



Contents lists available at ScienceDirect

## Journal of Economic Dynamics &amp; Control

journal homepage: [www.elsevier.com/locate/jedc](http://www.elsevier.com/locate/jedc)

# Origins of monetary policy shifts: A New approach to regime switching in DSGE models<sup>☆</sup>

Yoosoon Chang<sup>a,\*</sup>, Junior Maih<sup>b</sup>, Fei Tan<sup>c</sup><sup>a</sup> Department of Economics, Indiana University, USA<sup>b</sup> Norges Bank and BI Norwegian Business School, Norway<sup>c</sup> Department of Economics, Chaijetz School of Business, Saint Louis University, USA; Center for Economic Behavior and Decision-Making, Zhejiang University of Finance and Economics, China

## ARTICLE INFO

## Article history:

Received 19 March 2021

Revised 27 July 2021

Accepted 30 August 2021

Available online 10 September 2021

## JEL classification:

E52

C13

C32

## Keywords:

Monetary policy

DSGE Model

Regime switching

Latent autoregressive regime factor

Endogenous feedback

Expectations formation effects

## ABSTRACT

We examine monetary policy shifts by taking a new approach to regime switching in a small scale DSGE model with threshold-type switching in the monetary policy rule. The policy response to inflation is allowed to switch endogenously between two regimes, hawkish and dovish, depending on whether a latent regime factor crosses a threshold level. Endogeneity stems from the historical impacts of structural shocks driving the economy on the regime factor. By estimating our DSGE model using the U.S. data, we quantify the endogenous feedback from each structural shock to the regime factor to understand the sources of the observed policy shifts. This new channel sheds new light on the interaction between policy changes and measured economic behavior.

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## 1. Introduction

In time series analysis, there is a long tradition in modeling structural change as the outcome of a regime switching process [Hamilton (1988, 1989)]. By introducing an unobserved discrete-state Markov chain governing the regime in place, this class of models affords a tractable framework for the empirical analysis of time-varying dynamics that is endemic to many economic and financial phenomena.<sup>1</sup>

Despite the popularity of the Markov switching approach, its dynamics are ultimately governed by a regime switching process that is exogenous. This is especially unsatisfactory if we seek to truly understand the nature of policymaking and

<sup>☆</sup> This paper should not be reported as representing the views of Norges Bank. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank.

\* Corresponding author.

E-mail address: [yoosoon@indiana.edu](mailto:yoosoon@indiana.edu) (Y. Chang).

<sup>1</sup> Among recent developments of this approach, Kim (1994) made an important extension to the state space representation of dynamic linear models amenable to classical inference, whereas Chib (1996) presented a full Bayesian analysis for finite mixture models based on Gibbs sampling. An introductory exposition and overview of the related literature can be found in the monograph by Kim and Nelson (1999).

its impact on economic phenomena. As argued in [Chang et al. \(2017\)](#), the presence of endogeneity in regime switching is indeed ubiquitous<sup>2</sup> and, if ignored, may yield substantial biases and significantly deteriorate the precision in model parameter estimates. It follows that a more desirable approach to modeling occasional but recurrent regime shifts would admit some form of endogenous feedback from the behavior of underlying economic fundamentals to the regime generating process [[Diebold et al. \(1994\)](#), [Chib and Dueker \(2004\)](#), [Kim \(2004, 2009\)](#), [Kim et al. \(2008\)](#), [Bazzi et al. \(2014\)](#), [Kang \(2014\)](#), [Kalliovirta et al. \(2015\)](#), [Kim and Kim \(2018\)](#), among others].

The purpose of this paper is to introduce a threshold-type endogenous regime switching framework into dynamic stochastic general equilibrium (DSGE) models. Over the past 20 years, DSGE models have become a useful tool for quantitative macroeconomic analysis in both academia and policymaking institutions. One particularly important development is the effort to incorporate the possibility of exogenous regime shifts (e.g., changes in monetary policy) into the model specification. By making policy regime shifts endogenous, our framework allows for a greater scope for understanding the complex interaction between policy changes and measured economic behavior.

Following [Chang et al. \(2017\)](#), an essential feature of our model is that the monetary policy regime alternates between two regimes, hawkish and dovish, depending on whether an autoregressive latent factor crosses some threshold level. In our approach, two sources of random innovations jointly drive the latent factor and hence the policy regime change: (i) the internal innovations that represent the fundamental shocks inside the model; (ii) an external innovation that captures all other shocks left outside the model. The relative importance of the former source determines the degree of endogeneity in regime changes. The autoregressive nature of the latent factor, on the other hand, makes such endogenous effects long-lasting. Most importantly, regime switching of this type renders the transition probabilities time-varying in that they are all functions of the model's fundamentals. In the special case where regime shifts are purely driven by the external innovation, our model becomes observationally equivalent to one with conventional Markov switching.

The key contribution of this paper is to provide a framework within which we study the origins of monetary policy shifts. Ever since the seminal work of [Clarida et al. \(2000\)](#), modeling the time-varying behavior of monetary policy has remained an active research agenda for macroeconomists. While regime switching has emerged as a promising approach to modeling the time variation in monetary policy, scant attention in the literature has been paid to the macroeconomic origins that give rise to monetary policy shifts over time. Our paper takes a first step toward filling in this important gap—the aim here is not to identify policy switches that standard approaches would miss but rather to shed light into the reasons why the switching occurs.

Due to the substantial improvement in model fit, a multitude of empirical studies have proposed to estimate regime-switching DSGE models [[Schorfheide \(2005\)](#), [Liu et al. \(2011\)](#), [Bi and Traum \(2012, 2014\)](#), [Bianchi \(2013\)](#), [Davig and Doh \(2014\)](#), [Bianchi and Ilut \(2017\)](#), [Bianchi and Melosi \(2017\)](#), [Best and Hur \(2019\)](#), among others]. We complement the recent literature on likelihood-based estimation of DSGE models with exogenous Markov switching by making regime changes endogenous. At the core of our analysis is the endogenous feedback effect of underlying structural shocks on the regime generating process. As a result, economic agents update their beliefs each period about future regimes conditional on the realizations of shocks disturbing the economy. For pedagogical purposes, [Section 2](#) employs a simple model adopted from [Chang et al. \(2018b\)](#) to endogenize regime switching in monetary policy, which admits analytical characterizations of the mechanism at work. [Section 3](#) extends the simple model to a prototypical new Keynesian DSGE model, and derives its state space form that can be analyzed with our endogenous-switching Kalman filter. We find that price markup shocks play an important role in triggering the historical regime changes in the postwar U.S. monetary policy. To the best of our knowledge, modeling and quantifying such endogenous feedback channel are novel in the literature.

An important precursor to our study is [Davig and Leeper \(2006a\)](#), who applied a projection method to solve and calibrate a new Keynesian model where monetary policy rule changes whenever its target variables (e.g., inflation and output gap) cross some thresholds.<sup>3</sup> More recently, [Guerrieri and Iacoviello \(2015, 2017\)](#) developed piece-wise linear solution toolkit and likelihood-based estimation method for DSGE models subject to an occasionally binding constraint (e.g., the zero lower bound on nominal interest rates). In their setup, each state of the constraint—slack or binding—is handled as one of two distinct regimes under the same model. Like these studies, one may argue that it is more natural to assign the immediate triggers of regime switch to the state variables rather than the structural shocks. In this regard, our regime switching is indeed endogenous with respect to shocks. However, linking the regime factor directly to shocks is technically appealing because the dynamics of all state variables are ultimately driven by a small number of structural shocks. Moreover, the ultimate goal of macroeconomic policy is to stabilize the economy by mitigating the impacts of various shocks. A prominent example is the 1970s oil crisis when the U.S. experienced severe petroleum shortages with elevated prices, and this oil price shock serves the dual role of adverse supply disturbance and potential trigger of monetary policy to a more aggressive regime in the 1980s. We therefore view the structural shocks that generate aggregate fluctuations as the macroeconomic

<sup>2</sup> This endogeneity can be illustrated for instance by the fact that central banks switch to unconventional policies when the policy rate becomes constrained by the zero lower bound. Another example relates to mortgagors being subject to stringent borrowing conditions either when credit growth has been excessive or when there is a downturn in the economy. [Binning and Maih \(2017\)](#) present a general framework for modeling occasionally-binding constraints using regime switching. More recently, [Benigno et al. \(2020\)](#) apply similar techniques to document endogenous switches into and out of financial crises in Mexico.

<sup>3</sup> Using the same solution method, [Bi and Traum \(2012, 2014\)](#) estimated a real business cycle model where the government partially defaults on its debt whenever the debt level rises beyond a 'fiscal limit'.

origins of regime shifts, and establish a novel feedback channel by which they contribute to regime switching. Our analytical and empirical examples illustrate how the underlying structural shocks impact agents' expectations formation and monetary policy regimes through this endogenous feedback channel.

## 2. Analytical example

We first consider the simple frictionless model of inflation determination studied in Davig and Leeper (2006a), comprising a standard Fisher relation and an interest rate rule for monetary policy. We solve the model based on the assumption that economic agents, when forming rational expectations about future endogenous variables, exogenous shocks, and regime states, can observe their current and all past realizations. The regime factor, however, remains *latent* to agents as well as econometricians.<sup>4</sup> As will be shown subsequently, it merely serves as an auxiliary variable that rationalizes the specific functional forms of time-varying transition probabilities. Consequently, the model can be solved without reference to the regime factor. From an empirical perspective, though, it is interesting to extract the latent regime factor from the data, which may be correlated with measured economic behavior in a meaningful way.<sup>5</sup> In what follows, all variables are written in terms of log-deviations from their steady state values.

### 2.1. The setup

First, given a perfectly competitive endowment environment with flexible prices and one-period nominal bonds, the Fisher relation arises from the bond pricing equation and is, in its linearized form, given by

$$i_t = \mathbb{E}_t \pi_{t+1} + \mathbb{E}_t r_{t+1} \tag{1}$$

where  $i_t$  denotes the short-term nominal interest rate,  $\pi_t$  the inflation rate between periods  $t - 1$  and  $t$ , and  $\mathbb{E}_t$  the conditional expectation given information available through period  $t$ . The real interest rate  $r_t$  evolves as an autoregressive process

$$r_t = \rho_r r_{t-1} + \sigma_r \epsilon_{r,t}, \quad \epsilon_{r,t} \sim \mathbb{N}(0, 1) \tag{2}$$

where  $0 \leq \rho_r < 1$  and  $\sigma_r > 0$ .

Second, the monetary authority follows an interest rate feedback rule that systematically varies its response to contemporaneous inflation depending on the underlying policy regime, which is given by

$$i_t = \phi_s \pi_t + \sigma_i \epsilon_{i,t}, \quad \phi_s = \phi_0(1 - s_t) + \phi_1 s_t, \quad \epsilon_{i,t} \sim \mathbb{N}(0, 1) \tag{3}$$

where  $1 < \phi_0 < \phi_1$  and  $\sigma_i > 0$ .<sup>6</sup> Here the response of policy rate to inflation is allowed to switch between, in the spirit of Leeper's (1991) terminology, 'more active' and 'less active' monetary regimes. The regime index evolves according to  $s_t = \mathbb{1}\{w_t \geq \tau\}$  and the autoregressive regime factor follows  $w_t = \alpha w_{t-1} + v_t$ . Nevertheless, our approach and its supporting software is not bound by the assumption of only two regimes. By introducing either multiple regime factors or threshold levels, the model can switch among more than two regimes.

Finally, we introduce an endogenous feedback channel from the current structural shocks to the future regime changes. There are two standardized shocks—the real rate shock  $\epsilon_{r,t}$  and the monetary policy shock  $\epsilon_{i,t}$ —driving this simple economy, but for illustration purposes, we only consider the feedback from the current monetary policy shock  $\epsilon_{i,t}$  through its potential correlation with the next period regime factor innovation  $v_{t+1}$ . That is,

$$\begin{pmatrix} \epsilon_{i,t} \\ v_{t+1} \end{pmatrix} \sim \mathbb{N} \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right), \quad -1 < \rho < 1 \tag{4}$$

where  $\rho = \text{corr}(\epsilon_{i,t}, v_{t+1})$  is a correlation parameter that measures the strength of endogeneity in regime switching. The above specification is simple enough to admit an analytical solution, yet rich enough to highlight the general features of a rational expectations model with endogenous regime change in monetary policy.

It follows from (4) that

$$w_{t+1} = \alpha w_t + \rho \epsilon_{i,t} + \sqrt{1 - \rho^2} \eta_{t+1}, \quad \eta_{t+1} \sim \mathbb{N}(0, 1) \tag{5}$$

where the internal innovation  $\epsilon_{i,t}$  and the external innovation  $\eta_{t+1}$  are orthogonal to each other. This alternative representation of the regime factor points to the endogenous feedback from monetary interventions to the regime generating

<sup>4</sup> Instead, we assume that agents always know which regime they are in. However, they need to compute the transition probabilities in order to make decisions. In the exogenous switching case, these probabilities are constant. Clearly, it is hard to believe that monetary policy would randomly switch between different states irrespective of the state of the economic system. In our model, these probabilities are a function of the history of shocks and the law of motion for the regime factor process.

<sup>5</sup> For instance, using the same regime switching approach as in this paper, Chang et al. (2019) estimated a reduced-form model of monetary-fiscal regime changes and found that fiscal variables, particularly the tax to GDP ratio and net interest payment to government spending ratio, are among the most important variables in explaining the monetary regime factor.

<sup>6</sup> For analytical tractability, we assume that the monetary authority does not respond to output gap.

process. For example, when  $\epsilon_{i,t}$  and  $v_{t+1}$  are orthogonal (i.e.,  $\rho = 0$ ), regime shifts become the outcome of an exogenous process driven entirely by the non-structural shock  $\eta_{t+1}$ ; as  $\rho$  approaches one in absolute value, current monetary shocks bear more directly on future regime factor; when  $|\rho| = 1$ , future regimes turn out to depend only on monetary shock in the current period. In general, one would expect  $0 < |\rho| < 1$ .<sup>7</sup> The autoregressive coefficient  $\alpha$ , on the other hand, determines the persistency and hence the expected duration of each regime—as  $\alpha$  takes values towards positive (negative) unity, the model will on average undergo less (more) frequent regime shifts.

### 2.2. Transition probability

Like any regime switching model, it is essential to compute the associated transition probabilities. From a modeling point of view, it can be helpful to treat the regime factor as a computational device that produces the specific functional forms of transition probabilities adopted by this paper. To see that, first note

$$\mathbb{P}(w_{t+1} < \tau | w_t, \epsilon_{i,t}) = \mathbb{P}\left(\eta_{t+1} < \frac{\tau - \alpha w_t - \rho \epsilon_{i,t}}{\sqrt{1 - \rho^2}} \mid w_t, \epsilon_{i,t}\right) = \Phi_\rho(\tau - \alpha w_t - \rho \epsilon_{i,t})$$

where  $\Phi_\rho(x) = \Phi(x/\sqrt{1 - \rho^2})$ . Moreover,  $w_t$  is independent of  $\epsilon_{i,t}$  and follows  $\mathbb{N}(0, 1/(1 - \alpha^2))$ . Therefore, we can obtain the transition probability of staying in regime-0 (i.e., the less active regime) between periods  $t$  and  $t + 1$  explicitly as

$$\begin{aligned} p_{00}(\epsilon_{i,t}) &= \mathbb{P}(s_{t+1} = 0 | s_t = 0, \epsilon_{i,t}) \\ &= \frac{\mathbb{P}(w_{t+1} < \tau, w_t < \tau | \epsilon_{i,t})}{\mathbb{P}(w_t < \tau)} \\ &= \frac{\int_{-\infty}^{\tau} \int_{-\infty}^{\tau - \alpha x / \sqrt{1 - \alpha^2} - \rho \epsilon_{i,t}} \Phi_\rho(\tau - \alpha x / \sqrt{1 - \alpha^2} - \rho \epsilon_{i,t}) p_{\mathbb{N}}(x|0, 1) dx}{\Phi(\tau \sqrt{1 - \alpha^2})} \\ &= \frac{\int_{-\infty}^{\frac{\tau - \rho \epsilon_{i,t}}{\sqrt{1 - \rho^2}}} \int_{-\infty}^{\tau \sqrt{1 - \alpha^2}} p_{\mathbb{N}}(x, y | \mu_0, \Sigma_0) dx dy}{\Phi(\tau \sqrt{1 - \alpha^2})} \end{aligned} \tag{6}$$

where  $\mu_0 = [0, 0]'$ ,  $\Sigma_0 = [1, c; c, 1 + c^2]$ , and  $c = \alpha/(\sqrt{1 - \rho^2}\sqrt{1 - \alpha^2})$ . Analogously, the transition probability from regime-1 (i.e., the more active regime) in period  $t$  to regime-0 in period  $t + 1$  can be computed as

$$\begin{aligned} p_{10}(\epsilon_{i,t}) &= \mathbb{P}(s_{t+1} = 0 | s_t = 1, \epsilon_{i,t}) \\ &= \frac{\mathbb{P}(w_{t+1} < \tau, w_t \geq \tau | \epsilon_{i,t})}{\mathbb{P}(w_t \geq \tau)} \\ &= \frac{\int_{\tau}^{\infty} \int_{-\infty}^{\tau - \alpha x / \sqrt{1 - \alpha^2} - \rho \epsilon_{i,t}} \Phi_\rho(\tau - \alpha x / \sqrt{1 - \alpha^2} - \rho \epsilon_{i,t}) p_{\mathbb{N}}(x|0, 1) dx}{1 - \Phi(\tau \sqrt{1 - \alpha^2})} \\ &= \frac{\int_{-\infty}^{\frac{\tau - \rho \epsilon_{i,t}}{\sqrt{1 - \rho^2}}} \int_{-\infty}^{-\tau \sqrt{1 - \alpha^2}} p_{\mathbb{N}}(x, y | \mu_1, \Sigma_1) dx dy}{1 - \Phi(\tau \sqrt{1 - \alpha^2})} \end{aligned} \tag{7}$$

where  $\mu_1 = [0, 0]'$  and  $\Sigma_1 = [1, -c; -c, 1 + c^2]$ . Accordingly, we have  $p_{01}(\epsilon_{i,t}) = 1 - p_{00}(\epsilon_{i,t})$  and  $p_{11}(\epsilon_{i,t}) = 1 - p_{10}(\epsilon_{i,t})$ . Finally, the integrals in (6)–(7) can be easily evaluated using the cumulative bivariate normal distribution function. See the Online Appendix for derivation details.

In sum, our endogenous feedback mechanism renders the transition probabilities, which now become an integral part of the model solution, time-varying because they are all functions of  $\epsilon_{i,t}$ . A key difference from Chang et al. (2017), though, lies in that their  $\epsilon_{i,t}$  corresponds to a univariate regression error that can be readily computed given the data and parameter values, whereas ours represents structural shocks whose values remain latent to the econometrician and hence must be inferred from the data. In the special case of  $\rho = 0$ , transition probabilities (6)–(7) become constants and our model reduces to one with conventional Markov switching.

Fig. 1 plots transition probabilities as functions of monetary policy shock at selected values of  $(\alpha, \tau, \rho)$ , which is intended to highlight the distinct role of each parameter in shaping these functions while holding other parameters fixed. Overall,  $(\alpha, \tau)$  uniquely determine the constant levels of  $(p_{00}, p_{11})$  under exogenous Markov switching (solid and dash-dotted lines) owing to their one-to-one correspondence. Specifically, increasing the value of  $\alpha$  from  $-0.9$  to  $0.9$ , for example, raises the likelihood of remaining in the current regime by making the regime factor more persistent. Meanwhile, decreasing the value of  $\tau$  from  $0$  to  $-1$  favors the more active regime by making it relatively easier for the regime factor to stay above the threshold. On the other hand, the endogeneity parameter  $\rho$  introduces shock-specific variations into  $(p_{00}, p_{11})$  (dashed line). Due to the positive feedback effect ( $\rho = 0.9$ ), for instance, a one-time unanticipated tightening  $\epsilon_{i,t} > 0$  (loosening  $\epsilon_{i,t} < 0$ ) of

<sup>7</sup> From now on, we dispense with  $|\rho| = 1$  because predetermined regimes, though theoretically possible, make less economic sense.

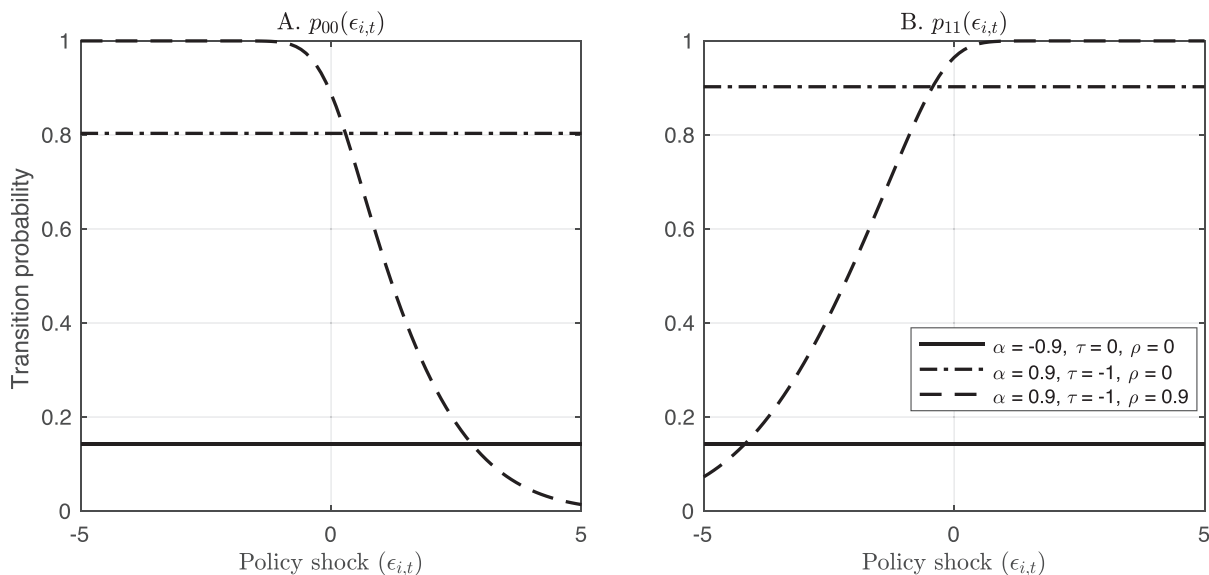


Fig. 1. Transition probabilities as functions of  $\epsilon_{i,t}$  at selected values of  $(\alpha, \tau, \rho)$ .

policy today increases the probability of staying in or shifting to the systematically tighter (looser) policy in the next period. Of course, the overall shapes of transition probability functions rest on all three parameters.

### 2.3. Equilibrium characteristics

Together with the transition probabilities (6)–(7), equations (1)–(3) constitute a nonlinear rational expectations system in the endogenous variables  $(\pi_t, i_t)$  that is driven by the exogenous variables  $(r_t, \epsilon_{i,t})$ . Substituting (2) and (3) into (1) delivers a regime-specific expectational difference equation in inflation

$$\phi_{s_t} \pi_t + \sigma_i \epsilon_{i,t} = \mathbb{E}_t \pi_{t+1} + \rho_r r_t \tag{8}$$

Now solving the model entails mapping the minimum set of state variables  $(r_t, \epsilon_{i,t})$  into the endogenous variable  $(\pi_t)$ . We find such a minimum state variable solution with the method of undetermined coefficients by postulating a regime-specific solution that takes an additive form

$$\pi_t = A_{s_t} r_t + B_{s_t} \epsilon_{i,t} \tag{9}$$

Chang et al. (2018b) show that these coefficients can be expressed as

$$A_{s_t} = \frac{\rho_r (\phi_1 - \phi_0) p_{s_t,0}(\epsilon_{i,t}) + \phi_1 (\frac{\phi_0}{\rho_r} - \mathbb{E} p_{00}(\epsilon_{i,t})) + \phi_0 \mathbb{E} p_{10}(\epsilon_{i,t})}{\phi_{s_t} (\phi_1 - \rho_r) (\frac{\phi_0}{\rho_r} - \mathbb{E} p_{00}(\epsilon_{i,t})) + (\phi_0 - \rho_r) \mathbb{E} p_{10}(\epsilon_{i,t})}, \quad B_{s_t} = -\frac{\sigma_i}{\phi_{s_t}} \tag{10}$$

where  $A_{s_t}$  also depends on  $\epsilon_{i,t}$  and the unconditional expectations  $\mathbb{E} p_{00}(\epsilon_{i,t})$  and  $\mathbb{E} p_{10}(\epsilon_{i,t})$  are constant terms.

Two special cases arise from the general solution (10). When regime changes are purely exogenous (i.e.,  $\rho = 0$ ), (10) reduces to

$$A_{s_t} = \frac{\rho_r (\phi_1 - \phi_0) p_{s_t,0} + \phi_1 (\frac{\phi_0}{\rho_r} - p_{00}) + \phi_0 p_{10}}{\phi_{s_t} (\phi_1 - \rho_r) (\frac{\phi_0}{\rho_r} - p_{00}) + (\phi_0 - \rho_r) p_{10}}, \quad B_{s_t} = -\frac{\sigma_i}{\phi_{s_t}} \tag{11}$$

where  $p_{s_t,0}$  and hence  $A_{s_t}$  become independent of  $\epsilon_{i,t}$ . Further imposing the restriction  $\phi_0 = \phi_1 = \phi > 1$  gives the equilibrium inflation under fixed regime

$$A_{s_t} = \frac{\rho_r}{\phi - \rho_r}, \quad B_{s_t} = -\frac{\sigma_i}{\phi} \tag{12}$$

where both  $A_{s_t}$  and  $B_{s_t}$  are non-random constants. It is clear from (12) that a more aggressive monetary stance (i.e., higher  $\phi$ ) can effectively insulate inflation against exogenous disturbances.

In all cases, we have  $A_{s_t} > 0$  so that a positive real rate shock, as it does in a fixed regime, raises the contemporaneous demand for consumption and thus inflation. With endogenous feedback in regime change, it immediately follows from (10) that two distinct effects on inflation emerge after a monetary policy intervention. First, conditioning on the prevailing policy regime, a monetary contraction tends to curtail inflation through its linear and direct effect captured by the  $B_{s_t} < 0$

term. Second, and more importantly, a monetary contraction also generates an *endogenous expectations-formation effect*—the difference between the impacts of a shock when the regime switches endogenously and when it switches exogenously—that is captured by the transition probability  $p_{s_t,0}(\epsilon_{i,t})$  in the  $A_{s_t}$  term. This nonlinear effect arises in that the intervention induces a change in agents' beliefs about the future policy regime. The resultant adjustment in agents' behavior can shift the projected path and probability distribution of equilibrium inflation in economically meaningful ways. As opposed to the endogenous switching case, such forward-looking effect vanishes in (11) under exogenous switching and thus there is no channel by which monetary interventions can alter agents' expectations about future regime.

To make the analytics more concrete, Fig. 2 isolates the endogenous expectations-formation effect by comparing the contemporaneous responses of inflation to exogenous shocks under endogenous ( $\rho = 0.9$ ) and exogenous switching ( $\rho = 0$ ). We consider a policy process that adjusts nominal rate only 'mildly' in the less active regime ( $\phi_0 = 1.1$ ), but to a degree more consistent with the standard Taylor rule specification when the more active regime is in place ( $\phi_1 = 1.5$ ). We also have the regime factor relatively persistent ( $\alpha = 0.9$ ) and somewhat favor the more active regime ( $\tau = -1$ ).<sup>8</sup> Panel B illustrates a slice of the impulse response surface reported in Panel A for a given positive real rate  $r_t = 0.5\%$ . Starting with the less active policy, contractionary monetary shocks trigger a positive feedback effect with endogenous switching, which leads agents to revise their beliefs towards a tighter policy in the subsequent period (as evinced by Fig. 1). In comparison with the exogenous switching case, this shift in expectations about future policy helps to further mitigate the inflationary effect of a positive real rate on impact. Analogously, expansionary monetary shocks trigger a negative feedback effect that bolsters agents' beliefs in the looser policy for the next period. Although less noticeable, the same positive real rate thus has larger impacts on current inflation.<sup>9</sup>

Formally, we measure expectations-formation effects from a policy intervention based on conditional inflation forecasts along the lines of Leeper and Zha (2003). Since private agents can observe current and all past realizations of endogenous variables  $(\pi_t, i_t)$ , exogenous shocks  $(r_t, \epsilon_{i,t})$ , and regime states  $s_t$ , they formulate rational expectations about future inflation based on the information set  $\mathcal{F}_T = \sigma(\{\pi_t, i_t, r_t, \epsilon_{i,t}, s_t\}_{t=0}^T)$ . Let  $I_T$  be a hypothetical intervention at period  $T$ , specified as a  $K$ -period sequence of exogenous policy actions  $I_T = \{\epsilon_{i,T+1}, \dots, \epsilon_{i,T+K}\}$ . Given the analytical solution under endogenous switching (10), it is straightforward to evaluate the forecast of  $\pi_{T+K}$  conditional on  $I_T$  as

$$\mathbb{E}[\pi_{T+K}|I_T, \mathcal{F}_T] = (p_{s_T,0}A_0(\epsilon_{i,T+K}) + p_{s_T,1}A_1(\epsilon_{i,T+K}))\rho_r^K r_T + (p_{s_T,0}B_0 + p_{s_T,1}B_1)\epsilon_{i,T+K}$$

where  $p_{s_T, s_{T+K}}$  for  $s_{T+K} = 0, 1$  are the  $K$ -period-ahead transition probabilities that depend on the sequence  $\{\epsilon_{i,T}, \dots, \epsilon_{i,T+K-1}\}$ . Likewise, forecasts under exogenous switching  $\mathbb{E}[\pi_{T+K}|I_T, \mathcal{F}_T, \rho = 0]$  and fixed regime  $\mathbb{E}[\pi_{T+K}|I_T, \mathcal{F}_T, s_t = 0, t = T + 1, \dots, T + K]$  can be computed using solutions (11) and (12), respectively. The total *expectations-formation effect*—a term coined by Leeper and Zha (2003)—refers to the difference between the impacts of a policy intervention when regime can and cannot switch, respectively

$$\text{Total effect} \equiv \mathbb{E}[\pi_{T+K}|I_T, \mathcal{F}_T] - \mathbb{E}[\pi_{T+K}|I_T, \mathcal{F}_T, s_t = 0, t = T + 1, \dots, T + K] \tag{13}$$

We focus on its endogenous component which is more relevant in our context

$$\text{Endogenous effect} \equiv \mathbb{E}[\pi_{T+K}|I_T, \mathcal{F}_T] - \mathbb{E}[\pi_{T+K}|I_T, \mathcal{F}_T, \rho = 0] \tag{14}$$

Of course, the difference between (13) and (14) quantifies the exogenous component.

For illustration purpose, we suppose the monetary authority undertakes a one-year contractionary intervention  $I_T = \{3, 2, 1, 0.5, 0, \dots, 0\}$  with gradually declining magnitudes from 3 to 1/2 standard deviations  $\sigma_i$  of the monetary policy shock. Panel A of Fig. 3 records the conditional forecasts of inflation obtained from the three models.<sup>10</sup> As expected, the intervention further tempers the initial inflationary impact of  $r_T = 0.5\%$  by about 1% with endogenous switching and 0.7% with exogenous switching relative to the fixed regime baseline.<sup>11</sup> These discrepancies among the forecasts translate into sizable expectations-formation effects in Panel B, which slowly taper off over the 10-year forecasting horizon. More importantly, over 30% of the total effect during the intervention can be attributed to its endogenous component (see Panel C). At very short horizons, the consecutive spikes in policy rate put forth a significantly positive feedback effect with endogenous switching, placing nearly all probability weight on the more active regime (see Panel D). Once the intervention ends after one year, the probability of remaining in the less active regime eventually converges to the long-run level, which is lower relative to the exogenous switching case under the current parameter setting.

### 3. Empirical illustration

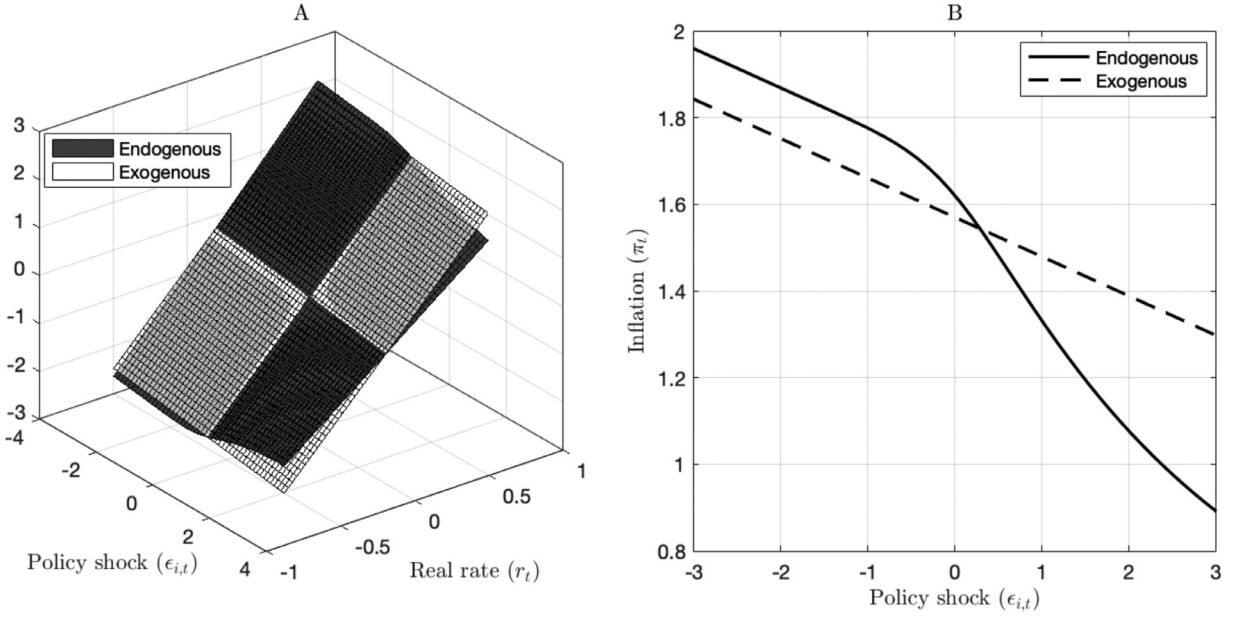
We now consider a small-scale new Keynesian DSGE model based on An and Schorfheide (2007) with the following features: a representative household and a continuum of monopolistically competitive firms; each firm produces a differ-

<sup>8</sup> The implied transition probabilities under exogenous switching are given by  $(p_{00}, p_{11}) = (0.8, 0.9)$ . See Fig. 1 for a visualization of the transition probability functions associated with endogenous switching.

<sup>9</sup> The analysis with a negative real rate is similar and therefore omitted here to conserve space.

<sup>10</sup> In the special case of one-period intervention  $I_T = \{1, 0, \dots, 0\}$ , these forecasts correspond to the conventional impulse response analysis.

<sup>11</sup> Similar to Davig and Leeper (2006a), our modeling of endogenous switching monetary policy is preemptive in that shocks realized at  $T$  shift agents' beliefs about the unknown regime at  $T + 1$  through their impacts on the regime factor  $w_{T+1}$ . This preemptive feature is absent when regimes switch exogenously. Because inflation rises by much less under endogenous switching, a preemptive policy behavior increases the effectiveness of monetary policy.



**Fig. 2.** Impulse response functions for inflation. Notes: Parameter settings under endogenous switching are  $(\phi_0, \phi_1) = (1.1, 1.5)$ ,  $(\rho_r, \sigma_r, \sigma_\pi) = (0.9, 0.1, 0.1)$ , and  $(\alpha, \tau, \rho) = (0.9, -1, 0.9)$ . Responses under exogenous switching are obtained by setting  $\rho = 0$  while keeping other parameters unchanged. The current regime is set to be less active, i.e.,  $s_t = 0$ .

entiated good and faces nominal rigidity in terms of quadratic price adjustment cost; a cashless economy with one-period nominal bonds; a monetary authority that controls nominal interest rate as well as a fiscal authority that passively adjusts lump-sum taxes to ensure its budgetary solvency; a labor-augmenting technology that induces a stochastic trend in consumption and output.

### 3.1. The setup

The model's equilibrium conditions in terms of the detrended variables can be summarized as follows. First, the household's optimizing behavior implies

$$1 = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\tau_c} \frac{R_t}{\gamma Z_{t+1} \pi_{t+1}} \right] \tag{15}$$

where  $0 < \beta < 1$  is the discount factor,  $\tau_c > 0$  the coefficient of relative risk aversion,  $c_t$  the detrended consumption,  $R_t$  the nominal interest rate,  $\pi_t$  the inflation between periods  $t - 1$  and  $t$ ,  $z_t$  an exogenous shock to the labor-augmenting technology that grows on average at the rate  $\gamma$ , and  $\mathbb{E}_t$  represents the conditional expectation given information available at time  $t$ . The firm's optimal price-setting behavior yields

$$1 = \frac{1 - c_t^{\tau_c}}{\nu_t} + \phi (\pi_t - \pi) \left[ \left( 1 - \frac{1}{2\nu_t} \right) \pi_t + \frac{\pi}{2\nu_t} \right] - \phi \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\tau_c} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \pi) \pi_{t+1} \right] \tag{16}$$

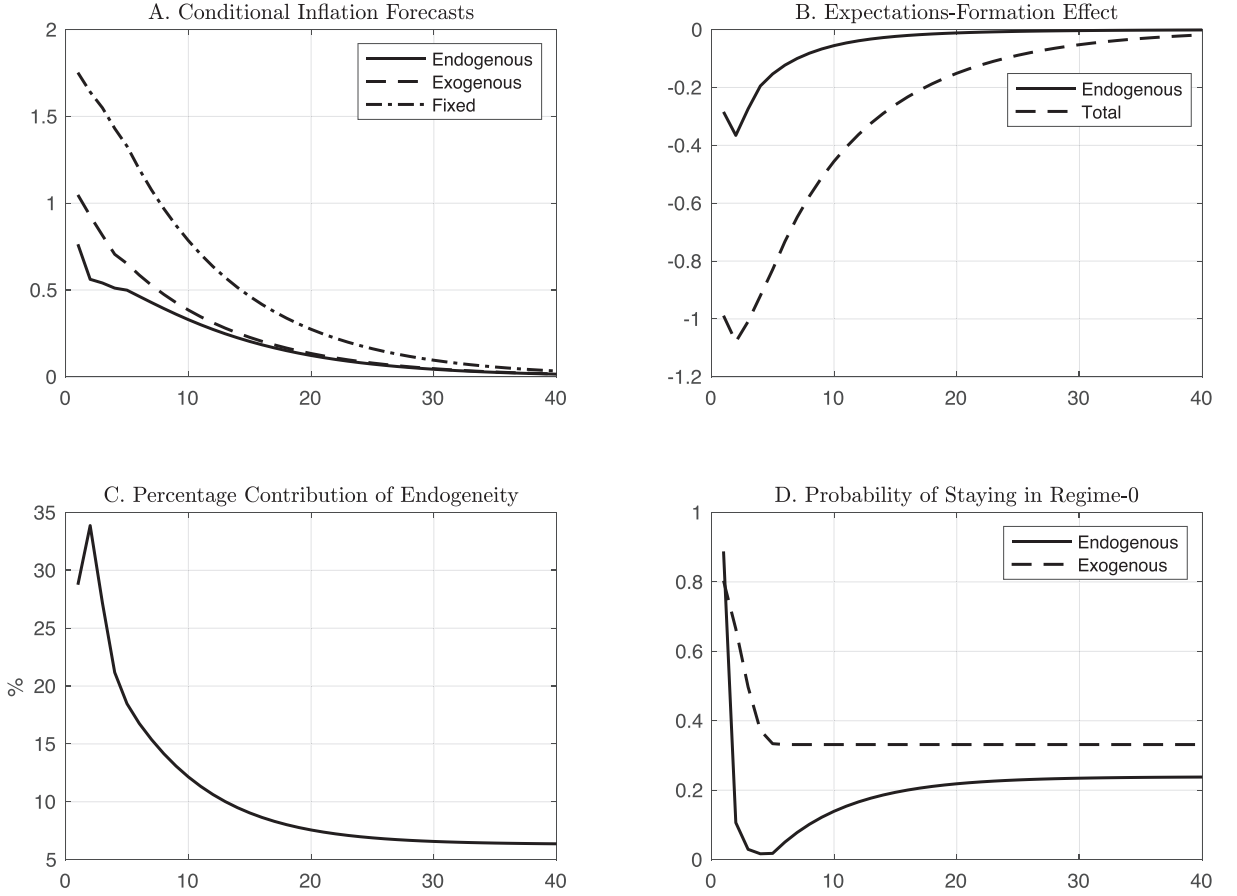
where  $\phi$  is the degree of price stickiness that relates to the slope of the so-called new Keynesian Phillips curve  $\kappa$  via  $\phi = \tau_c(1 - \nu)/(v\pi^2\kappa)$ ,  $\pi$  the steady state inflation,  $1/\nu$  the steady state of the demand elasticity  $1/\nu_t = u_t/(u_t - 1)$ ,  $u_t$  an exogenous price markup shock with the steady state  $u$ , and  $y_t$  the detrended output. The goods market clearing condition is given by

$$y_t = c_t + \left( 1 - \frac{1}{g_t} \right) y_t + \frac{\phi}{2} (\pi_t - \pi)^2 y_t \tag{17}$$

where  $g_t$  is an exogenous government spending shock with the steady state  $g$ .

Second, the monetary authority follows an interest rate feedback rule that reacts to deviations of inflation from its steady state and output from its potential value

$$R_t = R_t^{*1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R \epsilon_{R,t}}, \quad R_t^* = R \left( \frac{\pi_t}{\pi} \right)^{\psi_\pi} \left( \frac{y_t}{y_t^*} \right)^{\psi_y} \tag{18}$$



**Fig. 3.** Expectations-formation effects on inflation. Notes: Panel A plots the inflation forecasts conditional on the intervention  $I_T = \{3, 2, 1, 0.5, 0, \dots, 0\}$ . Panel B shows the difference between the total and endogenous expectations-formation effects. Panel C reports the ratio of endogenous to total effects. Panel D plots the  $K$ -period-ahead probabilities of remaining in the less active regime. The current set of state variables is set to  $(r_T, \epsilon_{i,T}, s_T) = (0.5\%, 0, 0)$ . See Fig. 2 notes for parameter settings.

where  $0 \leq \rho_R < 1$  is the degree of interest rate smoothing,  $\sigma_R > 0$ ,  $R$  the steady state nominal interest rate,  $\psi_\pi > 0$  and  $\psi_y > 0$  the policy rate responsive coefficients,  $y_t^* = (1 - \nu_t)^{1/\tau_c} g_t$  the detrended potential output that would prevail in the absence of nominal rigidities (i.e.,  $\phi = 0$ ), and  $\epsilon_{R,t}$  an exogenous policy shock.

Finally, each of  $(\ln z_t, \ln u_t, \ln g_t)$  evolves as an autoregressive process

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z \epsilon_{z,t} \tag{19}$$

$$\ln u_t = (1 - \rho_u) \ln u + \rho_u \ln u_{t-1} + \sigma_u \epsilon_{u,t} \tag{20}$$

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \epsilon_{g,t} \tag{21}$$

where  $0 \leq \rho_z, \rho_u, \rho_g < 1$  and  $\sigma_z, \sigma_u, \sigma_g > 0$ . The model is driven by the four innovations  $(\epsilon_{z,t}, \epsilon_{u,t}, \epsilon_{g,t}, \epsilon_{R,t})$  that are serially uncorrelated, independent of each other at all leads and lags, and normally distributed with zero mean and unit standard deviation.

There has been ample empirical evidence of time variation in estimated monetary policy rules documented in the literature. To keep the illustration simple and concrete, we allow the response of policy rate to inflation deviations to switch between more active and less active (or possibly ‘passive’) monetary policy regimes

$$\psi_\pi(s_t) = \psi_{\pi,0}(1 - s_t) + \psi_{\pi,1}s_t, \quad 0 \leq \psi_{\pi,0} < \psi_{\pi,1} \tag{22}$$

where the policy regime index evolves according to  $s_t = \mathbb{1}\{w_t \geq \tau\}$  and the regime factor follows  $w_t = \alpha w_{t-1} + \nu_t$ . To introduce the sources of endogeneity in regime change, we allow all current structural shocks to jointly influence the next period



regime through their correlations with the innovation  $v_{t+1}$ . That is,

$$\begin{pmatrix} \epsilon_t \\ v_{t+1} \end{pmatrix} \sim \mathbb{N} \left( \begin{pmatrix} 0_{4 \times 1} \\ 0 \end{pmatrix}, \begin{pmatrix} I_4 & \rho \\ \rho' & 1 \end{pmatrix} \right), \quad \rho' \rho < 1 \tag{23}$$

where  $\rho = [\rho_{zv}, \rho_{uv}, \rho_{gv}, \rho_{Rv}]' = \text{corr}(\epsilon_t, v_{t+1})$ . To keep the illustration simple and concrete, we abstract from time variation in the model structure other than that from the policy parameter  $\psi_\pi(s_t)$ .<sup>12</sup>

It follows from (23) that a more explicit representation of the regime factor can be written as

$$w_{t+1} = \alpha w_t + \underbrace{\rho_{zv}\epsilon_{z,t} + \rho_{uv}\epsilon_{u,t} + \rho_{gv}\epsilon_{g,t} + \rho_{Rv}\epsilon_{R,t}}_{\text{endogenous drivers}} + \underbrace{\sqrt{1 - \rho' \rho} \eta_{t+1}}_{\text{exogenous driver}}, \quad \eta_{t+1} \sim \mathbb{N}(0, 1) \tag{24}$$

where the internal innovations  $(\epsilon_{z,t}, \epsilon_{u,t}, \epsilon_{g,t}, \epsilon_{R,t})$  and the external innovation  $\eta_{t+1}$  are all orthogonal and have unit variance. Equation (24) asserts a complete separation between the four individual endogenous drivers  $(\epsilon_z, \epsilon_u, \epsilon_g, \epsilon_R)$  and the exogenous driver  $\eta$  of the regime factor. Also recall that  $(\rho_{zv}^2, \rho_{uv}^2, \rho_{gv}^2, \rho_{Rv}^2, 1 - \rho' \rho)$  measure the percentage contributions of  $(\epsilon_z, \epsilon_u, \epsilon_g, \epsilon_R, \eta)$  to the unconditional variance of  $w$  and hence the extents to which these drivers trigger historical regime changes. In what follows, we quantify how much of the U.S. monetary policy shifts can be attributed, respectively, to each of technology growth, price markup, government spending, and monetary policy shocks.

### 3.2. Solution method

The model is completed by deriving the probabilities governing regime transition from one period to the next. Analogous to the scalar case in the analytical model, the implied (endogenous) transition probabilities to the less active regime, which become an important part of the model solution, are given by

$$p_{00}(\epsilon_t) = \mathbb{P}(s_{t+1} = 0 | s_t = 0, \epsilon_t) = \frac{\int_{-\infty}^{\tau\sqrt{1-\alpha^2}} \Phi_\rho(\tau - \alpha x / \sqrt{1-\alpha^2} - \rho' \epsilon_t) p_{\mathbb{N}}(x|0, 1) dx}{\Phi(\tau\sqrt{1-\alpha^2})} \tag{25}$$

$$p_{10}(\epsilon_t) = \mathbb{P}(s_{t+1} = 0 | s_t = 1, \epsilon_t) = \frac{\int_{\tau\sqrt{1-\alpha^2}}^{\infty} \Phi_\rho(\tau - \alpha x / \sqrt{1-\alpha^2} - \rho' \epsilon_t) p_{\mathbb{N}}(x|0, 1) dx}{1 - \Phi(\tau\sqrt{1-\alpha^2})} \tag{26}$$

where  $\Phi_\rho(x) = \Phi(x/\sqrt{1-\rho' \rho})$ . See the Online Appendix for derivation details. When  $\epsilon_t$  and  $v_{t+1}$  are orthogonal (i.e.,  $\rho = 0_{4 \times 1}$ ), transition probabilities (25)–(26) become constants so that our model nests the exogenous Markov switching as a special case.

Equations (15)–(21) and (25)–(26) constitute a rational expectations system that can be cast into the generic form

$$\mathbb{E}[f_{s_t}(x_{t+1}, x_t, x_{t-1}, \epsilon_t) | \mathcal{F}_t] = 0 \tag{27}$$

where  $s_t = 1, \dots, h$  is the regime at time  $t$ ,  $f_{s_t}$  a vector of nonlinear functions,  $x_t$  a vector of model variables, and  $\epsilon_t$  a vector of shock innovations. As mentioned earlier, private agents can observe current and all past realizations of endogenous variables, exogenous shocks, and regime states, but not the regime factors. Accordingly, they formulate rational expectations about future variables on the basis of the information set  $\mathcal{F}_t = \sigma(\{x_k, \epsilon_k, s_k\}_{k=0}^t)$ .

System (27) has to be solved before the model can be taken to data. To that end, a spate of theoretical and empirical efforts have managed to solve regime-switching rational expectations models using numerical techniques. One strand of the literature embraces the projection method to iteratively construct policy functions over a discretized state space [Davig (2004), Davig and Leeper (2006a,b), Bi and Traum (2012, 2014), Davig et al. (2010, 2011), Richter et al. (2014)]. Nevertheless, global approximations suffer from, among other problems, the curse of dimensionality that renders the practical implementation computationally costly even for small-scale models. The second strand begins with a linear or linearized model as if its parameters were constant and then annexes Markov switching to certain parameters [Svensson and Williams (2007), Farmer et al. (2011), Bianchi (2013), Cho (2016), Bianchi and Ilut (2017), Bianchi and Melosi (2017)]. While this approach is not as limited by the model size as the projection method, linearization without accounting for the switching parameters may be inconsistent with the optimizing behavior of agents who are aware of the switching process in the original nonlinear model. The third strand circumvents the above problems by embedding regime switching in perturbation solutions whose accuracy can be enhanced with higher-order terms [Maih (2015), Foerster et al. (2016), Barthélemy and Marx (2017), Bjørnland et al. (2018), Maih and Waggoner (2018)].

We obtain the model solution to the nonlinear rational expectations system (27) using the perturbation approach of Maih and Waggoner (2018), which is more general than the ones found in the earlier literature. As opposed to Foerster et al. (2016), for example, it requires no partition of the switching parameters, allows for the possibility of multiple

<sup>12</sup> Initiated by Sims and Zha (2006), allowing for regime switching in both policy rules and shock volatilities has also come under scrutiny in DSGE models, but it would require introducing multiple regime factors in our setup, which is beyond the scope of this paper.

steady states and, more importantly, can handle models with endogenous transition probabilities.<sup>13</sup> To fix ideas, we seek a regime-specific policy function for  $x_t$  that depends on the minimum set of state variables  $(x_{t-1}, \epsilon_t)$

$$x_t = g_s(x_{t-1}, \epsilon_t) \tag{28}$$

Substituting (28) into (27) and integrating out the future regime yield

$$\mathbb{E} \left[ \sum_{j=1}^h p_{i,j}(\epsilon_t) f_i(g_j(x_{t-1}, \epsilon_t), \epsilon_{t+1}), g_i(x_{t-1}, \epsilon_t), x_{t-1}, \epsilon_t) \middle| \mathcal{F}_t \right] = 0 \tag{29}$$

where the notational convention that  $s_t = i$  and  $s_{t+1} = j$  is followed. In general, there is no analytical solution to (29) even when  $f_i$  is linear. Maih and Waggoner (2018) proposed to obtain a Taylor series approximation to (28) by introducing an auxiliary argument  $\chi$  (i.e., the perturbation parameter)

$$x_t = g_i(x_{t-1}, \epsilon_t, \chi) \tag{30}$$

that solves a perturbed version of (29)

$$\mathbb{E} \left[ \sum_{j=1}^h q_{i,j}(\epsilon_t, \chi) f_i(g_j(h_i(x_{t-1}, \epsilon_t, \chi), \chi \epsilon_{t+1}, \chi), g_i(x_{t-1}, \epsilon_t, \chi), x_{t-1}, \epsilon_t) \middle| \mathcal{F}_t \right] = 0 \tag{31}$$

with perturbed transition probability  $q_{i,j}(\epsilon_t, \chi)$  and perturbed policy function  $h_i(x_{t-1}, \epsilon_t, \chi)$ .

The principle of perturbation is to choose  $q_{i,j}(\epsilon_t, \chi)$  and  $h_i(x_{t-1}, \epsilon_t, \chi)$  such that (31) becomes the original system (29) when  $\chi = 1$ , but it reduces to a tractable and interpretable system when  $\chi = 0$ . While (31) is already successful in eliminating the stochastic disturbances when  $\chi = 0$ , choices of  $q_{i,j}(\epsilon_t, \chi)$  and  $h_i(x_{t-1}, \epsilon_t, \chi)$  still play a key role in interpreting the model's steady state  $x_i = g_i(x_i, 0, 0)$  around which the solution is expanded. In the context of exogenous Markov switching, Foerster et al. (2016) set  $q_{i,j}(\epsilon_t, \chi) = p_{i,j}$  and  $h_i(x_{t-1}, \epsilon_t, \chi) = g_i(x_{t-1}, \epsilon_t, \chi)$ . Plugging their choice into (31) and evaluating it at the steady state give

$$\sum_{j=1}^h p_{i,j} f_i(g_j(x_i, 0, 0), x_i, x_i, 0) = 0$$

Since the only point at which we know how to evaluate  $g_j$  is  $(x_j, 0, 0)$ , this implies that  $x_i = x_j$  for each  $j$  and hence the steady state must be independent of any regime.<sup>14</sup> By contrast, recognizing that switching parameters may imply distinct steady states, Maih and Waggoner's (2018) choice of

$$q_{i,j}(\epsilon_t, \chi) = \begin{cases} \chi p_{i,j}(\epsilon_t), & i \neq j \\ \chi(p_{i,i}(\epsilon_t) - 1) + 1, & i = j \end{cases}$$

and  $h_i(x_{t-1}, \epsilon_t, \chi) = g_i(x_{t-1}, \epsilon_t, \chi) + (1 - \chi)(x_j - x_i)$  implies

$$f_i(x_i, x_i, x_i, 0) = 0$$

Consequently,  $x_i$  can be readily interpreted as the deterministic steady state that would prevail in regime- $i$  when it is considered in isolation.

In practice, the solution algorithm of Maih and Waggoner (2018) has been coded up in RISE, a flexible object-oriented MATLAB toolbox developed by Junior Maih, that is well-suited for solving a general class of regime-switching DSGE models.<sup>15</sup> See the Online Appendix for a user guide.

### 3.3. Econometric method

We estimate the model with Bayesian methods using a common set of quarterly observations, ranging from 1954:Q3 to 2007:Q4: per capita real output growth (YGR), annualized inflation rate (INF), and effective federal funds rate (INT).<sup>16</sup> The actual data are constructed as in Appendix B of Herbst and Schorfheide (2015) and available from the Federal Reserve

<sup>13</sup> Barthélemy and Marx (2017) also generalized standard perturbation methods to solve a class of nonlinear rational expectations models with endogenous regime switching.

<sup>14</sup> In this regard, the literature typically defines the steady state as the one associated with the ergodic mean values of Markov switching parameters. Such unique steady state, however, need not be an attractor—a resting point towards which the model tends to converge in the absence of further shocks. For example, Aruoba et al. (2017) presented a model that exhibits two distinct attractors, i.e., a targeted-inflation steady state and a deflationary steady state.

<sup>15</sup> The toolbox is available, free of charge, at [https://github.com/jmaih/RISE\\_toolbox](https://github.com/jmaih/RISE_toolbox).

<sup>16</sup> Our sample begins when the federal funds rate data first became available and ends before the federal funds rate nearly hit its effective lower bound.

Economic Data (FRED). The observable variables are linked to the model variables through the following measurement equations

$$\begin{pmatrix} \text{YGR}_t \\ \text{INF}_t \\ \text{INT}_t \end{pmatrix} = \begin{pmatrix} \gamma^{(Q)} \\ \pi^{(A)} \\ \pi^{(A)} + r^{(A)} + 4\gamma^{(Q)} \end{pmatrix} + 100 \begin{pmatrix} \ln(z_t y_t / y_{t-1}) \\ 4 \ln(\pi_t / \pi) \\ 4 \ln(R_t / R) \end{pmatrix} \quad (32)$$

where  $(\gamma^{(Q)}, \pi^{(A)}, r^{(A)})$  are connected to the model's steady states via  $\gamma = 1 + \gamma^{(Q)}/100$ ,  $\beta = 1/(1 + r^{(A)}/400)$ , and  $\pi = 1 + \pi^{(A)}/400$ . Let  $\theta$  be a vector collecting all model parameters. In conjunction with the model solution, a first-order approximation to equations (32) and (30) around steady states can be cast into a nonlinear state space model.<sup>17</sup> To evaluate the associated likelihood function  $p(Y_{1:T}|\theta)$ , we adopt the endogenous-switching Kalman filter developed by Chang et al. (2018a), which is based on the algorithms of Kim (1994) and Chang et al. (2017). Calculations are simplified by an appropriate augmentation of the transition equation and exploiting the conditionally linear and Gaussian structure. Unlike simulation-based filters, this avoids sequential Monte Carlo integration and as such makes our filter computationally efficient. As a useful by-product of running the filter, the estimated autoregressive latent factor can be readily constructed from the filter outputs.<sup>18</sup> See the Online Appendix for technical details.

In the Bayesian paradigm, the likelihood obtained from the state space model is completed with a prior distribution  $p(\theta)$  summarizing the researcher's initial views of the model parameters. This prior information is updated with the sample information via Bayes' theorem

$$p(\theta|Y_{1:T}) \propto p(Y_{1:T}|\theta)p(\theta) \quad (33)$$

where the posterior distribution  $p(\theta|Y_{1:T})$  characterizing the researcher's updated parameter beliefs is calculated up to the normalization constant. Since the posterior surface can be highly irregular (e.g., non-elliptical and multimodal) in sophisticated DSGE models, we adopt the tailored randomized block Metropolis-Hastings (TaRB-MH) algorithm of Chib and Ramamurthy (2010) to sample the parameters from their joint posterior distribution. Two defining features of this algorithm, which are designed to overcome the difficulties in irregular problems, are worth mentioning in relation to the more standard posterior samplers (e.g., the random-walk MH algorithm) in empirical macroeconomics. One feature is the random clustering of the parameters into an arbitrary number of blocks at every iteration. Each block is then sequentially updated through an MH step. This proves to be particularly useful when the researcher does not have a priori knowledge about the correlation pattern of the parameters so that the grouping by correlation principle becomes infeasible. Another feature is the local tailoring of the proposal density to the location and curvature of the posterior distribution for a given block using an optimization routine. This allows for sizable moves from the neighborhood of the current parameter draw. To accelerate the TaRB-MH algorithm, we also explore an additional feature—blocking and tailoring at random frequencies instead of every iteration—advocated by Chib et al. (2021) that allows the researcher to obtain an efficient posterior sample within a reasonable amount of runtime.

### 3.4. Priors

Following standard practice, we assume that all structural parameters are a priori independent. Table 1 reports the priors of all structural parameters using 5% and 95% quantiles of the distributions. For the steady state parameters, the quarterly growth rate  $\gamma^{(Q)}$  follows a normal distribution with quantiles 0.14 and 0.79; the annualized inflation rate  $\pi^{(A)}$  follows a gamma distribution with quantiles 1.23 and 7.60; and the annualized real rate  $r^{(A)}$  follows a gamma distribution with quantiles 0.34 and 0.67. The priors on the structural shock processes are harmonized: the autoregressive coefficients  $(\rho_z, \rho_u, \rho_g, \rho_R)$  are beta distributed with quantiles 0.33 and 0.66, and the standard deviation parameters  $(\sigma_z, \sigma_u, \sigma_g, \sigma_R)$ , all scaled by 100, follow inverse-gamma type-I distribution with quantiles 0.08 and 1.12. Furthermore, the priors on the private sector parameters  $(\tau_c, \kappa, \nu, c/y)$  and the policy response to output  $\psi_y$  are largely adopted from An and Schorfheide (2007), whereas those on the policy responses to inflation  $(\psi_{\pi,0}, \psi_{\pi,1})$  closely follow the specification in Davig and Doh (2014), which a priori rule out the possibility of 'label switching' by imposing  $\psi_{\pi,0} < \psi_{\pi,1}$  and hence achieve regime identification. Finally, turning to the parameters for the autoregressive regime factor, the prior on  $\alpha$  centers at a rather persistent value that, together with the prior mean of  $\tau$ , implies symmetric transition probabilities (0.85,0.85) under exogenous switching. On the other hand, the relatively diffuse priors on  $(\rho_{zv}, \rho_{uv}, \rho_{gv}, \rho_{Rv})$  centering at zero reflect an agnostic view about the sign and degree of endogeneity in regime switching.<sup>19</sup>

<sup>17</sup> The standard stability concept for constant-parameter linear rational expectations models does not extend to the regime switching case. Instead, following the lead of Svensson and Williams (2007) and Farmer et al. (2011) among others, we adopt the concept of mean square stability to characterize first-order stable solutions.

<sup>18</sup> The filter is developed for the econometrician to evaluate the model's likelihood function. The latent factor itself, however, is not (explicitly) filtered by the economic agents. Having observed the history of shocks, and knowing the structure of the latent factor, the economic agents proceed to compute the probability that a regime switch will occur, which helps them make the economic decisions.

<sup>19</sup> We use the non-standard beta distribution  $\mathbb{B}(-1, 1)$  to bound each correlation parameter within its natural support  $(-1, 1)$ . Moreover, we impose the constraint  $\rho_{zv}^2 + \rho_{uv}^2 + \rho_{gv}^2 + \rho_{Rv}^2 < 1$  so as to avoid making future regimes completely predetermined. Note that if  $X \sim \mathbb{B}(a, b)$ , then  $Y = (X - a)/(b - a) \sim \mathbb{B}(0, 1)$ .

**Table 1**  
Priors and Posteriors of DSGE Parameters.

Parameter	Prior			Posterior			
	Density	5%	95%	Mode	5%	95%	Ineff Factor
$\tau_c$	G	1.25	2.88	4.20	3.59	5.00	17.8
$\kappa$	G	0.06	0.38	0.30	0.19	0.44	23.5
$\psi_y$	G	0.17	0.96	0.24	0.08	0.45	8.4
$\rho_z$	B	0.33	0.66	0.48	0.37	0.57	8.0
$\rho_u$	B	0.33	0.66	0.89	0.87	0.91	10.3
$\rho_g$	B	0.33	0.66	0.54	0.38	0.70	2.8
$\rho_R$	B	0.33	0.66	0.76	0.73	0.79	11.6
$r^{(A)}$	G	0.34	0.67	0.47	0.33	0.63	3.0
$\pi^{(A)}$	G	1.23	7.60	3.54	2.89	3.86	4.7
$\gamma^{(Q)}$	N	0.14	0.79	0.45	0.35	0.53	3.3
$\nu$	B	0.03	0.19	0.08	0.02	0.16	5.2
$c/y$	B	0.65	0.97	0.92	0.74	0.99	8.8
$100\sigma_z$	IG	0.08	1.12	0.66	0.58	0.76	7.9
$100\sigma_u$	IG	0.08	1.12	2.20	1.63	2.65	27.9
$100\sigma_g$	IG	0.08	1.12	0.30	0.21	0.41	5.5
$100\sigma_R$	IG	0.08	1.12	0.25	0.23	0.28	6.4
$\psi_{\pi,0}$	G	0.84	1.17	0.85	0.76	0.96	9.2
$\psi_{\pi,1}$	G	1.60	2.42	1.77	1.51	2.03	7.8
$\alpha$	B	0.80	0.96	0.94	0.89	0.97	4.4
$\tau$	N	-0.82	0.82	-0.29	-0.83	0.43	4.9
$\rho_{zv}$	B	-0.66	0.66	-0.19	-0.53	0.24	7.1
$\rho_{uv}$	B	-0.66	0.66	-0.71	-0.84	-0.26	5.2
$\rho_{gv}$	B	-0.66	0.66	0.02	-0.54	0.56	4.1
$\rho_{Rv}$	B	-0.66	0.66	0.16	-0.14	0.40	3.5

Notes: The following abbreviations are used: Gamma distribution (G), Normal distribution (N), Beta distribution (B), and Inverse-Gamma type-I distribution (IG). The posterior mode is obtained using the BFGS quasi-Newton method (available as a MATLAB function `csminwel` written by Chris Sims). This procedure is repeated multiple times, each of which is initialized at a high density point out of a large number of prior parameter draws.

### 3.5. Posterior estimates

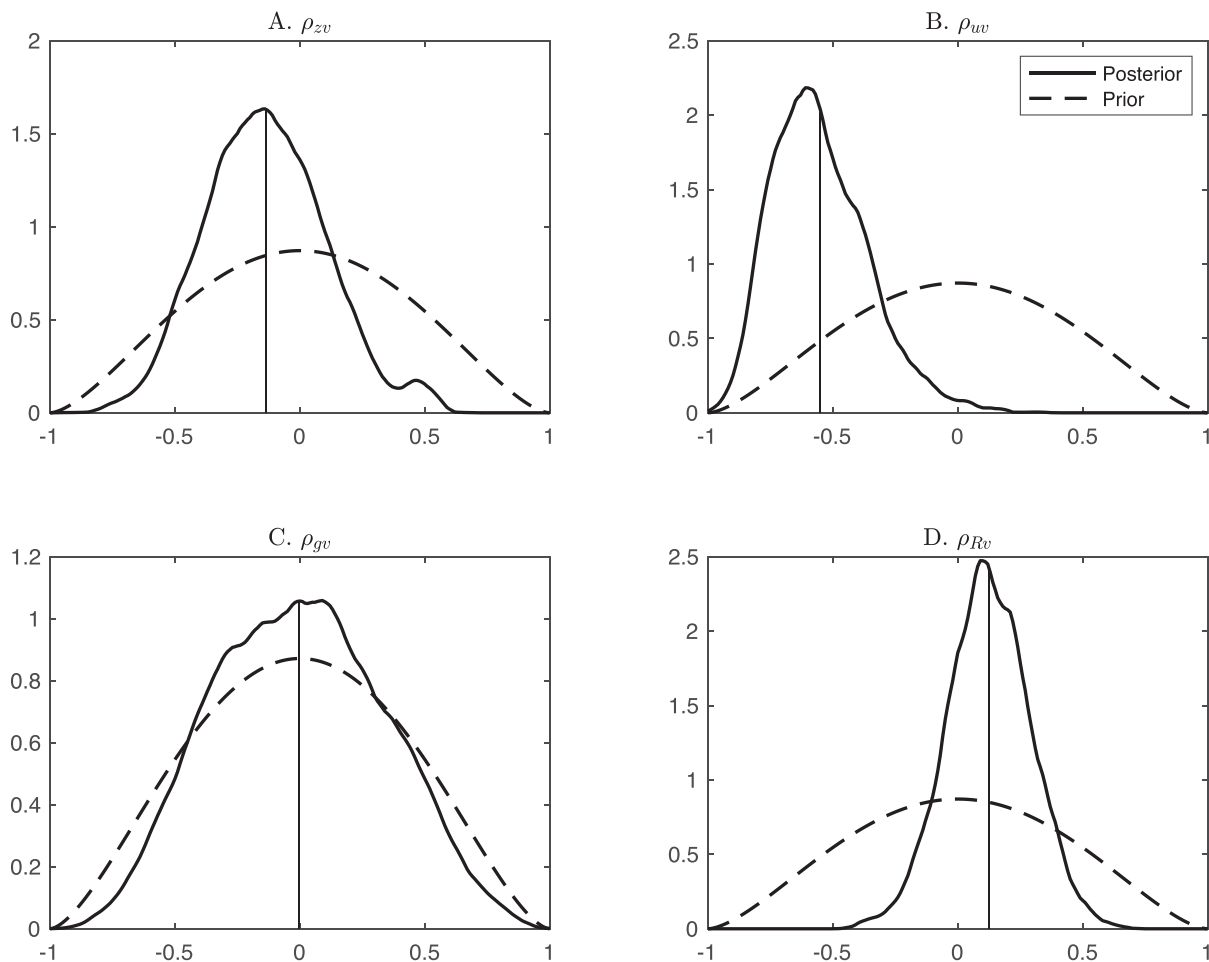
We sample a total of 11,000 draws from the posterior distribution using the TaRB-MH algorithm and discard the first 1000 draws as burn-in phase. The resulting 10,000 draws form the basis for performing our posterior inference. Owing to the efficiency gains achieved by the TaRB-MH algorithm, the required number of draws is substantially smaller than the number typically used for the conventional random-walk MH algorithm. Evidence of such gains can be seen from the very low inefficiency factors (ranging from 2.3 to 27.9, with most values below 10) in Table 1.<sup>20</sup> In conjunction with a rejection rate of approximately 50% in the MH step, the small inefficiency factors suggest that the Markov chain mixes well. We highlight several aspects of the posterior estimates reported in Table 1 as follows.

First, the 90% credible intervals show that the posterior distributions for most of the parameters are different from those implied by the prior. Hence, the data are informative about most of the estimated parameters. Second, turning to the switching parameters, there remains considerable cross-regime difference in the policy response to inflation, although the overall monetary policy stance appears to be somewhat less aggressive than assumed a priori—the 90% posterior intervals of  $(\psi_{\pi,0}, \psi_{\pi,1})$  are both shifted downward. Third, moving to the parameters  $(\alpha, \tau)$  unique to our threshold switching, a prior-posterior comparison reveals that the data support an even more persistent process for the regime factor, and slightly favor the more active regime by making it relatively easier for the regime factor to remain above the threshold. Finally, the posterior mode of  $\rho$  attributes a significant portion of regime developments to past price markup shocks (about 50%) and, to a lesser extent, technology growth shocks (about 4%) as well as monetary policy shocks (about 3%).<sup>21</sup>

Fig. 4 compares the prior and posterior densities of the correlation parameters  $(\rho_{zv}, \rho_{uv}, \rho_{gv}, \rho_{Rv})$ , which further substantiate the relevance of accounting for the endogenous feedback from historical macroeconomic shocks to the prevailing policy regime. Despite the diffuse priors, the data turn out to be informative to land the posteriors onto narrower areas of the parameter space that deliver tightly estimated degrees of endogeneity in regime switching. Most noticeable is the

<sup>20</sup> The inefficiency factor is defined as  $1 + 2 \sum_{j=1}^K w(j/K) \rho(j)$ , where we set the truncation parameter  $K = 200$  and weight the autocorrelation function  $\rho(\cdot)$  using the Parzen kernel  $w(\cdot)$ . An efficient sampler produces autocorrelations that decay to zero within a few lags and hence leads to low inefficiency factors.

<sup>21</sup> We do not separately estimate the exogenous switching case because it is nested as a special case of our approach (i.e., by imposing the parameter restriction  $\rho = 0_{4 \times 1}$ ). Although not reported in the paper, we find that the exogenous switching model yields comparable parameter estimates and a marginally smaller log marginal likelihood. The associated log likelihoods, however, can still be significantly different. To see this, let  $\Delta = \ln p(Y_{1:T} | \hat{\theta}) - \ln p(Y_{1:T} | \hat{\theta}, \rho = 0_{4 \times 1})$ , where  $\hat{\theta}$  denotes the posterior mode under each model, and compare twice the differential  $\Delta$  (10.99) to the 5% critical value of  $\chi^2$  limit distribution with four degrees of freedom (9.49). It follows that the data favor the model that accounts for the endogeneity in regime changes.



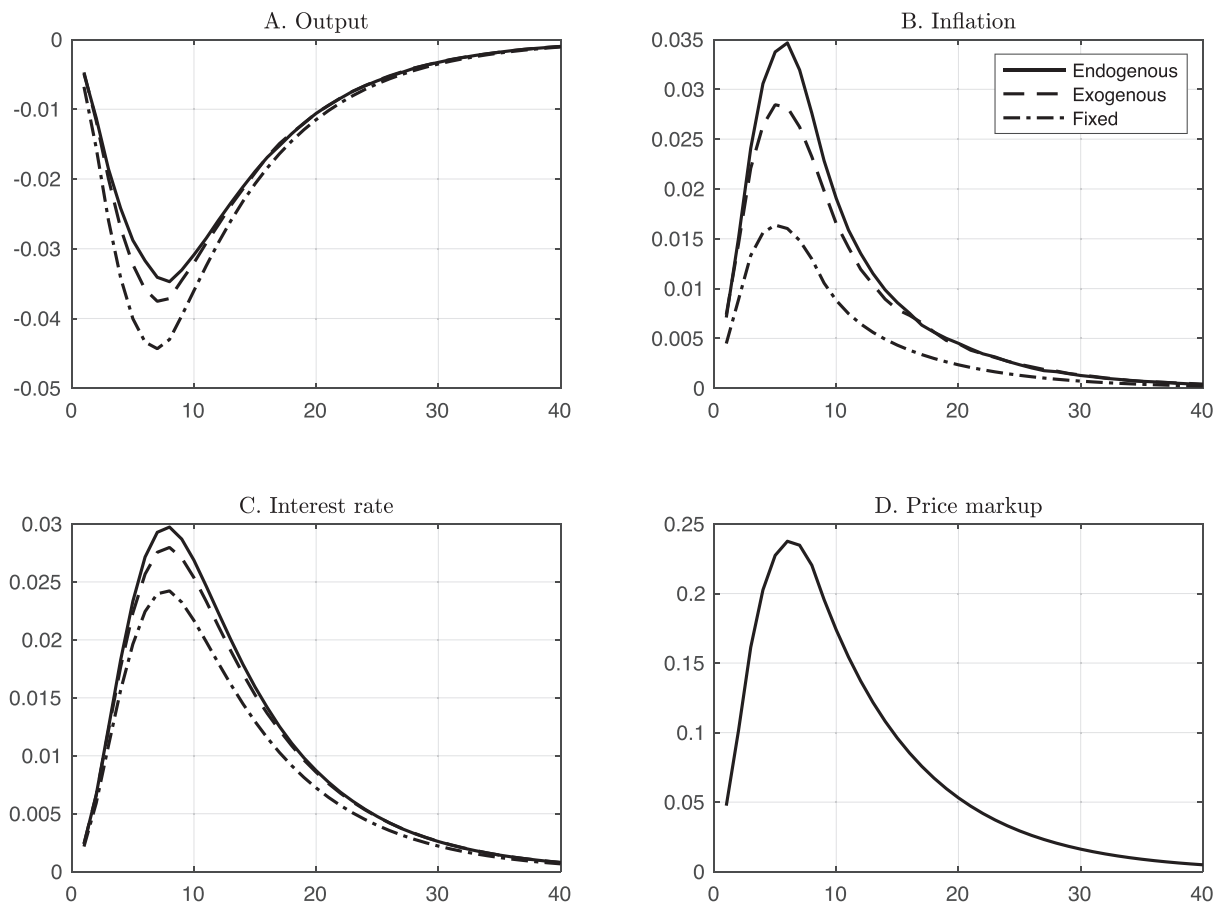
**Fig. 4.** Prior and posterior density functions of correlation parameters. Notes: The vertical lines delineate the posterior means. All posterior densities are estimated using the Epanechnikov kernel function.

endogenous feedback from price markup shock, whose posterior mass falls almost entirely on the negative territory. As a result, adverse supply disturbances unambiguously increase the likelihood of staying in or shifting to the less active regime, consistent with a countercyclical monetary policy that is ‘leaning against the wind’. Less evident is the posterior distribution of the endogenous feedback from technology growth shock that concentrates somewhat more on the negative territory. Thus, on balance, favorable technological advancements tend to make the less active regime more likely, suggesting an accommodative monetary policy to promote long-term economic growth. These patterns connect broadly to theoretical work and empirical observations about how central banks routinely act.<sup>22</sup> Government spending shock, on the other hand, plays no observable role in driving the regime changes since in this simple model it only affects output but not output gap (hence consumption, inflation, and nominal rate).

Just as the analytical example of Section 2 demonstrates, endogenizing regime changes here can generate important expectations-formation effects beyond what the exogenous switching can do. For example, consider a sequence of positive markup shocks  $\{2, 2.5, 3, 2.5, 2, 1.5, 1, 0.5, 0, \dots, 0\}$  that aims to mimic the sharp increases in oil prices beginning in 1973 (see Panel A of Fig. 6 below) and reproduces the dire inflationary scenario afterward. This exogenous “intervention” persists for two years and peaks in magnitude after two quarters since it hits the economy. Fig. 5 displays the conditional forecasts of key model variables for the three cases.<sup>23</sup> With the more active regime initially in place, higher price markups not only dampen aggregate supply and hence engender the usual stagflation—a mix of rising inflation (Panel B) and falling economic

<sup>22</sup> See Section 2 for an analysis of the endogenous feedback caused by monetary policy shock.

<sup>23</sup> Because the model is inherently nonlinear and does not admit an analytical solution, sequential Monte Carlo methods are required to numerically integrate out the regime index in each period. To compute the conditional forecasts under endogenous switching, for instance, we simulate  $N = 500$  trajectories of regime states  $\{s_{T+1}^{(i)}, \dots, s_{T+k}^{(i)}\}_{i=1}^N$  according to the time-varying transition probabilities (25)–(26), apply the regime-specific policy function (28) to obtain the corresponding model variables  $\{x_{T+1}^{(i)}, \dots, x_{T+k}^{(i)}\}_{i=1}^N$  conditional on a pre-specified sequence of markup shocks  $\{\epsilon_{u,T+1}, \dots, \epsilon_{u,T+k}\}$  but



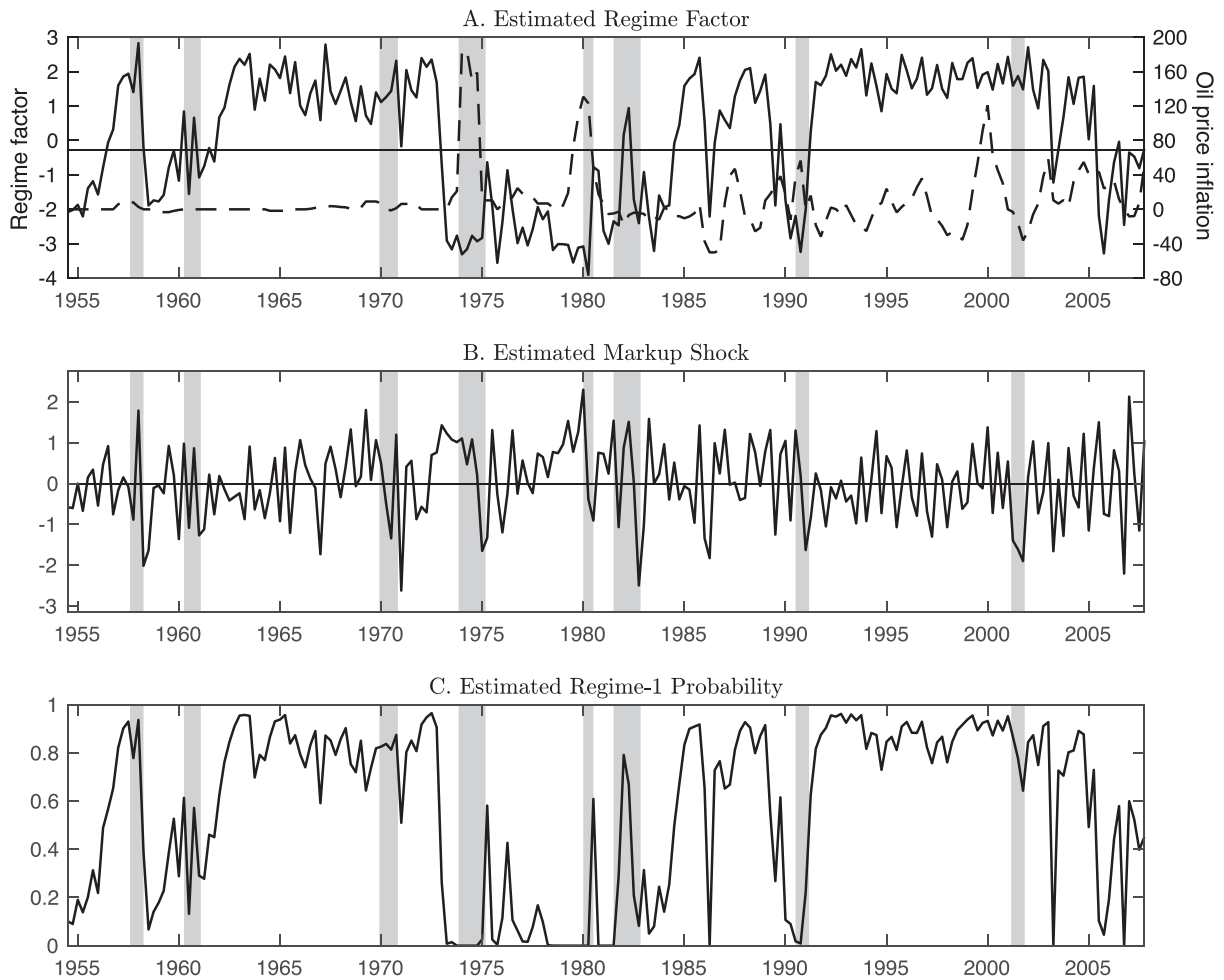
**Fig. 5.** Dynamic responses to price markup shocks at the posterior mode under endogenous switching. Notes: Panels A–C plot the projected paths of key endogenous variables conditional on a two-year sequence of price markup shocks  $\{2, 2.5, 3, 2.5, 2, 1.5, 1, 0.5, 0, \dots, 0\}$ . The exogenous-switching response is obtained by setting  $\rho_{zv} = \rho_{uv} = \rho_{gv} = \rho_{Rv} = 0$  while keeping other parameters unchanged. The current regime is set to be more active and held constant to compute the fixed regime response. All variables are in level deviations from steady state.

activity (Panel A)—as in the fixed regime, but also trigger a negative feedback effect ( $\rho_{uv} = -0.71$ ) that induces agents to form a stronger belief in the less active regime under which inflation rises by more and output falls by less. This shift in agents’ expectations spells out the discrepancies in the forecasts under exogenous and endogenous switching (i.e., with and without the restriction  $\rho_{uv} = 0$ ). In response to the inflationary pressure that outweighs the recessionary pressure, the monetary authority raises the nominal rate according to its policy rule (Panel C).

Unlike the Markov switching filter of Kim (1994), our filter produces an important by-product—an estimated time series of the regime factor  $w_{t|t}$  as portrayed in Panel A of Fig. 6—that complements the information contained in the estimated regime-1 probability  $p_{t|t}^1$  (Panel C). In particular, this series exhibits prolonged swings that identify the U.S. monetary policy as sluggishly fluctuating between the more active (i.e.,  $w_{t|t} \geq \tau$ ) and less active (i.e.,  $w_{t|t} < \tau$ ) regimes, the timing and nature of which are broadly consistent with the previous empirical findings. Such a pattern also aligns quite well with the narrative record of policymakers’ beliefs documented in Romer and Romer (2004): the more active stance of the late 1950s and most of the 1960s under chairman William McChesney Martin Jr. and of the mid-1980s and beyond under Paul Volcker and Alan Greenspan stemmed from the conviction that inflation has high costs and few benefits; the less active stance of most of the 1970s under Arthur Burns and G. William Miller stemmed from an overestimate of the natural rate of unemployment as well as an underestimate of the sensitivity of inflation to economic slack.

We conclude with a final remark on the regime factor. Loosely speaking, it conveniently packs, through a reduced-form mechanism, various structural and extraneous sources of regime changes into one single index that is likely to proxy for some observables relevant to the regime generating process. To corroborate this interpretation, Panel A of Fig. 6 relates the estimated regime factor to a common measure of oil price inflation. The apparent negative correlation since 1973 stems

zeroing out all other shocks, and take the sample average of  $\{x_t^{(i)}\}_{i=1}^N$  for  $t = T + 1, \dots, T + K$ . In the special case of one-period shock  $\{1, 0, \dots, 0\}$ , these forecasts are equivalent to the generalized impulse response functions of Koop et al. (1996).



**Fig. 6.** Filtered regime factor, markup shock, and regime-1 probability at the posterior mode under endogenous switching. Notes: The left and right vertical axes in Panel A measure the filtered autoregressive regime factor (solid line) and the oil price inflation (dashed line, constructed as the growth rate of Spot Crude Oil Price: West Texas Intermediate (WTI) [WTISPLC], retrieved from FRED), respectively. The horizontal line in Panel A delineates the threshold level. The policy regime is more (less) active when the regime factor is above (below) the threshold level. Shaded bars indicate recessions as designated by the National Bureau of Economic Research.

from the fact that the latter variable potentially identifies the price markup shock, which is found to be the most important structural driver of monetary regime changes according to our estimated DSGE model. For example, the precipitous run-ups in oil prices during the 1970s are well captured by the continuous upticks in the estimated markup shocks (Panel B).

#### 4. Concluding remarks

This paper aims at broadening the scope for understanding the complex interaction between recurrent regime changes and measured economic behavior. To that end, we introduce a threshold-type endogenous regime switching into the popular DSGE models. In our approach, regime changes are, through an autoregressive latent factor, jointly driven by the internal innovations that represent the fundamental shocks inside the model and an external innovation that captures all other shocks left outside of the model. This allows the behavior of underlying economic fundamentals to bear more directly on the regime generating process. When regime switches are purely driven by the external innovation, our model reduces to one with exogenous Markov switching.

As an application of our methodology, we estimate a prototypical new Keynesian DSGE model with threshold switching in monetary policy rule, and illustrate how the underlying structural shocks impact agents' expectations formation and monetary policy regimes through our novel endogenous feedback channel. We find compelling statistical support for the endogenous feedback from historical shocks to the prevailing policy regime.

We remind the reader that quantifying such feedback mechanism depends on nearly every aspect of private and policy behavior. A comprehensive investigation calls for a richly structured medium-scale DSGE model with fiscal details in both

the model specification and the observable data. Another natural extension of our framework is to permit multiple regimes and latent factors. We defer these extensions to a sequel to this paper.

## Acknowledgements

The authors would like to thank seminar and conference participants at Australian National University, Central University of Finance and Economics, Keio University, Korea Development Institute, MIT-Harvard, North Carolina State University, Renmin University of China, the Reserve Bank of Australia, Toulouse School of Economics, the University of California–Santa Barbara, the University of Chicago Booth School of Business, the University of Sydney, 2017 NBER Workshop on DSGE Models at Philadelphia Fed, 2018 Workshop on “Nonlinear Models in Macroeconomics and Finance for an Unstable World” at Norges Bank, 2018 International Association for Applied Econometrics Annual Conference at Montréal, 2018 Taiwan Econometric Society Annual Meeting, the 27th Annual Symposium of the Society for Nonlinear Dynamics and Econometrics at Dallas, and the 29th Annual Meeting of the Midwest Econometrics Group at Columbus, as well as Francesco Bianchi, Yongok Choi, Olivier Coibion, Luca Guerrieri, Edward Herbst, Eric Leeper, Chaojun Li, Joon Park, Shi Qiu, Daniel Waggoner, and Tao Zha for very useful and stimulating comments. Fei Tan acknowledges financial support from the Chaifetz School of Business summer research grant.

## Supplementary material

Supplementary material associated with this article can be found, in the online version, at [10.1016/j.jedc.2021.104235](https://doi.org/10.1016/j.jedc.2021.104235)

## References

- An, S., Schorfheide, F., 2007. Bayesian analysis of dsge models. *Econom Rev* 26 (2), 113–172.
- Aruoba, S.B., Cuba-Borda, P., Schorfheide, F., 2017. Macroeconomic dynamics near the zlb: a tale of two countries. *Rev Econ Stud* 85 (1), 87–118.
- Barthélemy, J., Marx, M., 2017. Solving endogenous regime switching models. *Journal of Economic Dynamics and Control* 77, 1–25.
- Bazzi, M., Blasques, F., Koopman, S.J., Lucas, A., 2014. Time-varying transition probabilities for markov regime switching models. *Journal of Time Series Analysis* 38 (3), 458–478.
- Benigno, G., Foerster, A.T., Otrok, C., Rebucci, A., 2020. Estimating Macroeconomic Models of Financial Crises: An Endogenous Regime-Switching Approach. Staff Reports. Federal Reserve Bank of New York.
- Best, G., Hur, J., 2019. Bad luck, bad policy, and learning? a markov-switching approach to understanding postwar u.s. macroeconomic dynamics. *European Economic Review*, forthcoming.
- Bi, H., Traum, N., 2012. Estimating sovereign default risk. *The American Economic Review: Papers & Proceedings* 102 (3), 161–166.
- Bi, H., Traum, N., 2014. Estimating fiscal limits: the case of greece. *Journal of Applied Econometrics* 29 (7), 1053–1072.
- Bianchi, F., 2013. Regime switches, agents’ beliefs, and post-world war ii u.s. macroeconomic dynamics. *Rev Econ Stud* 80 (2), 463–490.
- Bianchi, F., Ilut, C., 2017. Monetary/fiscal policy mix and agent’s beliefs. *Rev Econ Dyn* 26, 113–139.
- Bianchi, F., Melosi, L., 2017. Escaping the great recession. *American Economic Review* 107 (4), 1030–1058.
- Binning, A., Maih, J., 2017. Modelling Occasionally Binding Constraints Using Regime-Switching. Working Paper. Norges Bank.
- Bjørnland, H.C., Larsen, V.H., Maih, J., 2018. Oil and macroeconomic (in)stability. *American Economic Journal: Macroeconomics* 10 (4), 128–151.
- Chang, Y., Choi, Y., Park, J.Y., 2017. A new approach to model regime switching. *J Econom* 196 (1), 127–143.
- Chang, Y., Kwak, B., Qiu, S., 2019. U.s. monetary and fiscal policy regime changes and their interactions. Manuscript.
- Chang, Y., Tan, F., Wei, X., 2018a. State space models with endogenous regime switching. Manuscript.
- Chang, Y., Tan, F., Wei, X., 2018b. A structural investigation of monetary policy shifts. Manuscript.
- Chib, S., 1996. Calculating posterior distributions and modal estimates in markov mixture models. *J Econom* 75, 79–97.
- Chib, S., Dueker, M.J., 2004. Non-Markovian regime switching with endogenous states and time-varying state strengths. Working Papers. Federal Reserve Bank of St. Louis.
- Chib, S., Ramamurthy, S., 2010. Tailored randomized block mcmc methods with application to dsge models. *J Econom* 155 (1), 19–38.
- Chib, S., Shin, M., Tan, F., 2021. DSGE-SVT: An econometric toolkit for high-dimensional DSGE models with SV and  $t$  errors. forthcoming at *Computational Economics*.
- Cho, S., 2016. Sufficient conditions for determinacy in a class of markov-switching rational expectations models. *Rev Econ Dyn* 21, 182–200.
- Clarida, R., Galí, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. *Quarterly Journal of Economics* 115 (1), 147–180.
- Davig, T., 2004. Regime-switching debt and taxation. *J Monet Econ* 51 (4), 837–859.
- Davig, T., Doh, T., 2014. Monetary policy regime shifts and inflation persistence. *Rev Econ Stat* 96 (5), 862–875.
- Davig, T., Leeper, E.M., 2006. Endogenous Monetary Policy Regime Change. In: Reichlin, L., West, K.D. (Eds.), *NBER International Seminar on Macroeconomics 2006*. MIT Press, Cambridge, MA, pp. 283–302.
- Davig, T., Leeper, E.M., 2006. Fluctuating Macro Policies and the Fiscal Theory. In: Acemoglu, D., Rogoff, K., Woodford, M. (Eds.), *NBER Macroeconomics Annual 2006*, 21. MIT Press, Cambridge, pp. 247–298.
- Davig, T., Leeper, E.M., Walker, T.B., 2010. ‘Unfunded liabilities’ and uncertain fiscal financing. *J Monet Econ* 57 (5), 600–619.
- Davig, T., Leeper, E.M., Walker, T.B., 2011. Inflation and the fiscal limit. *Eur Econ Rev* 55 (1), 31–47.
- Diebold, F.X., Lee, J.-H., Weinbach, G., 1994. Regime Switching with Time-varying Transition Probabilities. In: Hargreaves, C. (Ed.), *Nonstationary Time Series Analysis and Cointegration*. Oxford University Press, pp. 283–302.
- Farmer, R.E., Waggoner, D.F., Zha, T., 2011. Minimal state variable solutions to markov-switching rational expectations models. *Journal of Economic Dynamics and Control* 35 (12), 2150–2166. *Frontiers in Structural Macroeconomic Modeling*
- Foerster, A., Rubio-Ramírez, J.F., Waggoner, D.F., Zha, T., 2016. Perturbation methods for markov-switching dynamic stochastic general equilibrium models. *Quant Econom* 7 (2), 637–669.
- Guerrieri, L., Iacoviello, M., 2015. Occbin: a toolkit for solving dynamic models with occasionally binding constraints easily. *J Monet Econ* 70, 22–38.
- Guerrieri, L., Iacoviello, M., 2017. Collateral constraints and macroeconomic asymmetries. *J Monet Econ* 90, 28–49.
- Hamilton, J.D., 1988. Rational-expectations econometric analysis of changes in regime: an investigation of the term structure of interest rates. *Journal of Economic Dynamics and Control* 12 (2), 385–423.
- Hamilton, J.D., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* 57 (2), 357–384.
- Herbst, E.P., Schorfheide, F., 2015. Bayesian estimation of DSGE models. Princeton University Press.



- Kalliovirta, L., Meitz, M., Saikkonen, P., 2015. A gaussian mixture autoregressive model for univariate time series. *Journal of Time Series Analysis* 36 (2), 247–266.
- Kang, K.H., 2014. Estimation of state-space models with endogenous markov regime-switching parameters. *Econom J* 17 (1), 56–82.
- Kim, C.-J., 1994. Dynamic linear models with markov-switching. *J Econom* 60 (1), 1–22.
- Kim, C.-J., 2004. Markov-switching models with endogenous explanatory variables. *J Econom* 122 (1), 127–136.
- Kim, C.-J., 2009. Markov-switching models with endogenous explanatory variables ii: a two-step mle procedure. *J Econom* 148 (1), 46–55.
- Kim, C.-J., Kim, J., 2018. Non-markovian regime-switching models. Manuscript.
- Kim, C.-J., Nelson, C.R., 1999. *State-Space models with regime switching*. MIT Press, Cambridge, MA.
- Kim, C.-J., Piger, J., Startz, R., 2008. Estimation of markov regime-switching regression models with endogenous switching. *J Econom* 143 (2), 263–273.
- Koop, G., Pesaran, M., Potter, S.M., 1996. Impulse response analysis in nonlinear multivariate models. *J Econom* 74 (1), 119–147.
- Leeper, E.M., 1991. Equilibria under 'active' and 'passive' monetary and fiscal policies. *J Monet Econ* 27 (1), 129–147.
- Leeper, E.M., Zha, T., 2003. Modest policy interventions. *J Monet Econ* 50 (8), 1673–1700.
- Liu, Z., Waggoner, D.F., Zha, T., 2011. Sources of macroeconomic fluctuations: aregime-switching dsge approach. *Quant Econom* 2 (2), 251–301.
- Maih, J., 2015. Efficient Perturbation Methods for Solving Regime-Switching DSGE Models. Working Paper. Norges Bank.
- Maih, J., Waggoner, D., 2018. Perturbation methods for dsge models with time-varying coefficients and transition matrices. Mimeograph, Norges Bank.
- Richter, A., Throckmorton, N., Walker, T., 2014. Accuracy, speed and robustness of policy function iteration. *Computational Economics* 44, 445–476.
- Romer, C.D., Romer, D.H., 2004. Choosing the federal reserve chair: lessons from history. *Journal of Economic Perspectives* 18 (1), 129–162.
- Schorfheide, F., 2005. Learning and monetary policy shifts. *Rev Econ Dyn* 8 (2), 392–419.
- Sims, C.A., Zha, T., 2006. Were there regime switches in us monetary policy? *American Economic Review* 96 (1), 54–81.
- Svensson, L.E.O., Williams, N., 2007. Monetary policy with model uncertainty: Distribution forecast targeting. CEPR Discussion Paper No. DP6331.