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# Effect of the migration mechanism based on risk preference on the evolution of cooperation



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# ABSTRACT

Individual migration is an effective means of promoting cooperation in the spatial structure. We can think of migration as a kind of risky investment, so risk attitudes can produce an effect on the migration decision. In order to understand the relationship between risk preference and the evolution of cooperation, the spatial prisoner's dilemma game model with individual migration based on risk preference is established. By introducing the homogeneity of risk preference, we find that lower risk aversion values keep a high level of cooperation under a larger defection parameter, while the cooperation level can be raised when the whole population is risk-seeking and at lower risk aversion values under a smaller defection parameter. Under the heterogeneous risk preference assumption, simulation results indicate that the cooperation strategy is a winning strategy in a steady state for a wide parameter space and the cooperation level decreases with increasing in the variance of risk preference. From typical snapshots, we can see that co-evolution of the network structure and cooperation strategy has been realized. Cooperative clusters can also be found in the typical snapshots, which have proved the migration mechanism based on risk to be effective in favoring the evolution of cooperation.

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## 1. Introduction

Cooperation is altruistic behavior which can increase the fitness of others but reduce their own fitness during evolution. If evolution is survival of the fittest, the cooperators will be eliminated through a process of natural selection. However, cooperation is a widespread phenomenon in social life. From an evolutionary perspective, how this behavior can evolve is a difficult question to explain. Many mechanisms have been proposed by scholars from various perspectives, such as economics [1,2], biology [3] and statistical physics [4-7], to resolve the evolution of cooperation. Kin selection, direct reciprocity, indirect reciprocity, multilevel selection and spatial selection are the five main mechanisms discussed by Nowak [8]. Kin selection is used for explaining the cooperative phenomenon with a genetic relationship between individuals; direct reciprocity is aimed at explaining the cooperation in the repeated game; indirect reciprocity is mainly reputation-based mechanisms; multilevel selection studies cooperation in view of group theory. However, the research perspectives mentioned above cannot explain the cooperative behavior in a one-shot prisoner's dilemma game. The evolution of cooperation based on the

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perspective of spatial selection has become an important research direction [9-16]. In such studies, evolutionary game theory is a theoretical tool [17,18]; network structure is a carrier [19-23]; computer simulation is the main research method [24,25].

Spatial selection is able to promote the emergency of cooperation because network structure provides favorable conditions for the formation of cooperative clusters [26]. Individual migration is an important mechanism that can promote the survival and even enlargement of cooperator clusters [27-35]. For example, the walk-away migration strategy [36,37], the success-driven migration [38], the always-move rule [39]. In addition, collective-risk social dilemma in the public game addresses the effect of self-organizing risk-driven migration on the evolution of cooperation [40-43]. Other migration mechanisms have also been used to discuss the evolution of cooperation [44-46]. Migration changes the partners of the game and the learning strategy, resulting in changes in the co-evolution of cooperation. The co-evolutionary model which allows strategy updating and network updating is closer to reality [47,48], such as the co-evolution based on reputation [49] and the co-evolution dependent on random mobility [44]. Individual migration is an important method for the formation of co-evolution and cooperative clumps.

Risk preference is defined as the attitudes toward facing risk, which is another important factor affecting the decision of migration. The effect of risk preference on the evolution of cooperation has been discussed, and the relationship between risk and cooperation is that risk averse players are beneficial for the emergency of cooperation [50-53]. However, the above research is discussed on a static network structure, and the migration mechanism based on risk preference types has rarely been investigated in previous studies. Because we can think of migration as a kind of risky investment, risk attitudes can produce an effect on the decision of migration. It is documented that risk seekers are more likely to migrate than risk averse players [54,55]. In reality, risk averse players, risk-seeking players and risk neutral players are the three main risk preference types.

In order to further improve the research on risk and cooperation, and understand the relationship between movementbased risk preference and the evolution of cooperation, referring to the study of Flache and Jaeger et al. [51,54], we establish such a model; in this model, whether an agent migrates depends on his risk preference style and his surroundings. To be specific, the migration rate of the agent shows positive correlation with payoff and negative correlation with migration cost, and the risk seeker has a higher migration rate than risk averse ones. The characteristic of this model considers not only objective factors-migration cost and the wealth of the individual but also subjective factors-risk preference styles of the individual. Then, can movement based on risk preference promote the emergency of cooperation? How does the heterogeneity of risk preference affect the cooperation level? In order to answer these questions, we consider individual migration based on the homogeneity of risk preference and the heterogeneity of risk preference in the spatial prisoner's dilemma game. The simulation results demonstrate that risk preference migration is an effective mechanism for promoting cooperative behavior under certain conditions.

The rest of the paper is organized as follows. In Section 2, we present the related network structure and Spatial Prisoner's Dilemma Game model with the introduction of migration based on risk preference. In Section 3, the simulation results and analysis of their implications are given. In Section 4, we summarize the main conclusions and discuss further research directions.

### 2. Model

We adopt a regular  $L \times L$  square lattice with periodic boundary conditions and Von Neuman neighborhood, where each site is either empty or occupied by an agent and the agents are uniformly distributed at random in the initial situation. We define  $\rho = N/L \times L$  as the density of the population, where N is the number of agents in the spatial structure and  $L \times L$  is the size of the square lattice. We consider an evolutionary Prisoners' dilemma game (PDG) in the following studies. In the evolutionary Prisoners' dilemma game, each agent *i*, designated either as cooperation(C) or defection(D) with equal probability, plays games with his neighbors and obtains the total payoff  $P_i$ , which depends on the payoff matrix A and the strategies of his neighbors. The payoff matrix A is given below:

$$\begin{array}{c} C & D \\ C \begin{pmatrix} R & S \\ D \begin{pmatrix} T & P \end{pmatrix} \end{array}$$

where *R* is the payoff for mutual cooperation, while *P* is the payoff for mutual defection. When a cooperator meets a defector, *S* is the payoff for the cooperator, and *T* is the payoff for the defector. Equalities T > R > P > S and 2R > T + S are the conditions for the PDG. We adopt the simple re-scaled payoff matrix: T = b, R = 1.0, P = .1 and S = 0.0 according to [22,26]. Besides, we want to point out one aspect here: according to [56,57], scaling parameters, Dg' and Dr', for the dilemma strength in our presumed PD games satisfies Dg' > 0 and Dr' = 0.1/0.9. In this case, the condition is not exactly the same as that of boundary games with pure chicken games.

In addition to the network structure and the evolutionary game model, the migration mechanism is another important part of our model. The migration mechanism considering strategy, migration costs and risk preference may reflect real-world setups to some extent. The rate of migration is given as follows.

$$\nu_{i} = \begin{cases} 0, & \text{if } N_{D} = 0\\ \left(\frac{P_{i}}{P_{i}+c}\right)^{2^{\alpha_{i}}}, & \text{if } N_{D} > 0 \end{cases}$$
(1)

If the neighbors of the agent are all cooperators  $(N_D = 0)$ , the agent cannot move; otherwise, the agent moves with a probability  $(P_i/(P_i + c))^{2^{\alpha_i}}$ , where *c* is the migration cost, and  $\alpha_i$  is a parameter responsible for the style of risk preference. The migration rate is positively related to his payoff and negatively related to his migration cost. The exponential function of risk preference and the base  $P_i/(P_i + c) < 1$  ensure the appropriate migration rate of the individual and the existence of risk preference heterogeneity in groups. When  $\alpha = 0$ , it means that the agent is absolutely risk neutral and when  $\alpha > 0$ , it represents that the risk attitude of the agent, so it indicates that the agent is risk seeking. The study by Jaeger et al. [54] shows that the parameter of the risk appetite  $\alpha_i$  follows Gaussian distributions, that is to say,  $\alpha_i \sim N(0, \sigma^2)$ , where the mean value 0 means the risk preference type is risk neutral on average and  $\sigma$  represents the heterogeneity of risk preference types are focused on risk neutral. Because  $\alpha_i \sim N(0, \sigma^2)$ , the probability density function of the random variable  $\alpha$  is:

$$p(\alpha) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\frac{\alpha^2}{2\sigma^2}\right\}$$
(2)

Overall, the Monte Carlo simulation procedure of our model consists of three elementary steps. First, a selected agent plays PDG with its neighbors and calculates the payoffs from the game until all agents finish it, except for the isolated agent whose payoff is set to be 0. Second, a randomly selected agent updates its strategy according to the unconditional imitation rules [47], which means that if the highest payoff of his neighbors is greater than the selected agents', the neighbor will pass his strategy to the agent. Next, the selected agent randomly migrates to an empty site according to formula (1). We assume the isolated agents can move to a randomly chosen neighboring site. We adopt an asynchronous fashion in the updating process. Lastly, each agent transforms his strategy to the opposite one with a certain probability called mutation rate *q*.

In general, we set up an individual migration model based on risk preference in the spatial prisoner's dilemma game, which is the epitome of simplicity for the social life to some extent and provides the possibility of studying the relationship between risk preference and cooperative behavior. The goal of this paper is to answer the following questions:

- 1. What is the requirement of parameters for a high cooperation level?
- 2. How do the migration cost and the heterogeneity of risk preference affect the cooperation level?
- 3. What is the evolutionary process for cooperative behavior from the point of view of the spatial pattern?
- 4. How has the network structure changed from the initial moment until the steady state?

To answer these questions, the fraction of cooperators  $f_c$ , which characterizes the cooperation level at the steady state, is a major variable we mainly focused on. We will provide further analysis on the effect of risk preference on the evolution of cooperation in Section 3.

#### 3. Results

#### 3.1. The homogeneity of risk preference

First, we study the simplest migration based on strategy and cost in the prisoner dilemma game with homogeneity of risk preference, which means that the risk preference is the same for all individuals in the group. Fig. 1 demonstrates the change in the fraction of cooperators when the risk preference is homogeneous. All individuals in the group are risk seeking, risk neutral and risk averse when  $\alpha < 0$ ,  $\alpha = 0$  and  $\alpha > 0$ , respectively. The smaller the  $\alpha$ , the stronger the risk seeking is. On the contrary, the risk aversion of the agent becomes stronger with the increase in  $\alpha$ .

It is observed from Fig. 1 that the performance of cooperative behavior is different for b = 1.2 and b = 1.4, especially for risk seeking. For b = 1.2, the fraction of cooperators is close to 1 when  $-10 < \alpha \le 0$ , independent of the migration cost c and the mutation rate q. However, the cooperation level declines from approximately 0.99 to 0.4 for q = 0 and  $0 < \alpha \le 10$ , while the fraction of cooperators declines to about 0.5 for q = 0.001 and  $0 < \alpha \le 10$ . Compared with the risk aversion population, the risk-seeking preference makes the cooperators have more opportunities to escape the defectors, resulting in the expansion of cooperator clumps.

For b = 1.4, when the population is risk seeking  $(-10 < \alpha < 0)$ , the cooperation level is near 0 for different levels of risk seeking except for  $\alpha = -2$  and  $\alpha = -1$  when c = 50. When the population is risk neutral, the cooperative phenomenon can survive and even keep a high level at c = 50. When the population is risk averse, the lower risk aversion values ( $\alpha = 1$ , 2, and 3) keep the high level of cooperation when c = 1. However, the cooperation level quickly decreases when over a certain risk aversion. Compared with the smaller defection parameter (b = 1.2), the larger b values make more cooperators turn into defectors. And the risk-seeking preference makes the defectors rapidly invade the rest of the cooperators, leading to extinction for the cooperators. Lower risk aversion results in a smaller migration rate for defectors. Thus, the rest of the cooperative clumps leads to the lower cooperation level.



**Fig. 1.** The effects of homogeneity of risk preference on the evolution of cooperation for different mutation rates q. (a) q = 0, (b) q = 0.001. Other parameters: L = 40,  $\rho = 0.75$ . Each data point is obtained after averaging over 20 independent runs, and the fraction of cooperators is obtained by averaging over 500 time steps after 9500 time steps for q = 0.0. The fraction of cooperators is obtained by averaging over 10,000 time steps after 100,000 time steps for q = 0.001.



**Fig. 2.** The process of the evolution of cooperation based on the heterogeneity of risk preference. (a) b = 1.2, (b) b = 1.4. Other parameters: q = 0.001,  $\rho = 0.75$ .

#### 3.2. The heterogeneity of risk preference

We analyze the influence of the homogeneity of risk preference on the evolution of cooperation in part 3.1. However, there are actually different risk preference styles in the whole population. Next, the heterogeneity of risk preference is assumed in the population and  $\alpha_i \sim N(0, \sigma^2)$ .

In order to roughly evaluate the role of the heterogeneity of risk preference on cooperation, we observe the evolutionary process of cooperation as time in the spatial prisoner's dilemma game, as depicted in Fig. 2. Three important results can be found: first, cooperative behavior can survive and even achieve a value above a 0.8 cooperation level at a lower defection parameter. Second, there is an enduring period and expanding period in the process of evolution. The cooperation level dramatically declines from an initial value of 0.5 in the enduring period, and the cooperation level appears to trend upward in the expanding period because of the role of the migration mechanism. Third, the cooperation level at b = 1.2 is smaller than b = 1.4. In summary, we can say that the migration mechanism based on the heterogeneity of risk preference is beneficial for the emergency of cooperation.

Next, we concretely study the fraction of cooperators as a function of the variance of risk preference distribution  $\sigma$  at different conditions. Observed from Fig. 3(a) and (b), the heterogeneity of risk preference sustains a high cooperation level at certain values and with  $\sigma$  increasing, and  $f_c$  decreases slightly when b = 1.2, except for the point at b = 1.2, q = 0.001 and c = 50. For b = 1.4 and c = 50, regardless of whether there is strategy mutation, the cooperation level shows a declining trend



**Fig. 3.** The effects of heterogeneity of risk preference on the evolution of cooperation for different mutation rates q. (a) q = 0, (b) q = 0.001. Other parameters: L = 40,  $\rho = 0.75$ . The data points are obtained in the same way as in Fig. 1.

apart from a few points, while the cooperation level fluctuates on a lower level for b = 1.4 and c = 1. When the risk appetite is heterogeneous in the group, there are six types of agents, including relative risk neutral cooperators, relative risk neutral defectors, risk-seeking cooperators, risk-seeking defectors, risk aversion cooperators and risk aversion defectors. Fig. 4 shows probability density distribution at different variance of risk preference  $\sigma$ . With increasing  $\sigma$ , the tail of the normal distribution becomes wider and wider (from (a) to (d)); thus, the larger the fraction for absolute risk seeking and risk aversion, the smaller the fraction of relative risk neutral. Therefore, with increasing  $\sigma$ , the migration rate of absolute risk seeking is increasing, while the migration rate of absolute risk aversion is decreasing. The higher migration rate for risk-seeking defectors leads to the invasion of cooperators, reducing the cooperation level. In general, compared with the homogeneous risk preference, the heterogeneous risk preference still keeps the emergence of cooperators and  $f_c$  decreases with  $\sigma$  increasing.

Observed from Fig. 3, we can see that the cooperation level at b = 1.4 and c = 50 is higher than at b = 1.4, c = 1. How does the migration cost affect the cooperation level?

Fig. 5 depicts the effect of migration cost on the level of cooperation. To observe clearly, we provide a large image in Fig. 5(b), (c), (e) and (f) for q = 0 and b = 1.2, q = 0 and b = 1.4, q = 0.001 and b = 1.2, q = 0.001 and b = 1.4, respectively. In Fig. 5, we can observe three results. First, compared with Fig. 5(a) and (d), we can see that the cooperation level at q = 0 is mostly higher than at q = 0.001. Second, the cooperation level at b = 1.2 is above 0.9 but slightly decreases from 0.98 to 0.91 with the increase in migration cost, where  $c \in (1, 50)$ , regardless of mutation rate q, observed from Fig. 5(b) and (e). Third,  $f_c$  increases with the increase in migration cost when b = 1.4 except for a few points at higher  $\sigma$  values, irrespective of the value of mutation rate q, which can be seen from Fig. 5(c) and (f).

For b = 1.2, because the migration rate of cooperators around the defectors is reduced owing to the increase in migration cost, the spread of cooperators is prohibited. However, because of the small defection parameter and the migration mechanism, the cooperation level remains above 0.9. For b = 1.4, due to the larger defection parameter, more cooperators turn into defectors in the initial evolution. And the decrease in migration rate for risk-seeking defectors as a result of the increase in migration cost reduces the chances for defectors to invade the cooperators. Therefore, it provides a beneficial environment for the expansion of cooperator clusters.

Through analysis of Figs. 3 and 5, we can see that the migration cost and the variance of the risk preference play an important role in the evolution of cooperation-based migration. Next, we provide an extensive analysis for the influence of *c* and  $\sigma$  on  $f_c$  in Fig. 6. The contour plot of the fraction of cooperators  $f_c$  for b = 1.2 and b = 1.4 are displayed in Fig. 6(a) and (b), respectively. Fig. 6(a) shows that  $f_c$  obtains a high cooperation level ( $f_c > 0.9$ ) in the whole range of simulation parameters. Moreover,  $f_c$  is near to 1 at the left-bottom of the image. We can also find that  $f_c$  can be increased with the decrease of *c* and  $\sigma$ . However, for a larger defection parameter (b = 1.4), Fig. 6(b) shows that the cooperation level fluctuates at b = 1.4. The highest cooperation level ( $f_c \approx 0.7$ ) can be found at the right-bottom of the image. The percentage of cooperators, however, is particularly low at c = 1, regardless of the value of variance  $\sigma$ . In general, certain heterogeneity of the risk preference in the population parameter values. Other 2D phase diagrams of  $\sigma(c)-\rho$  will be discussed in Figs. 10 and 12.

In order to further examine whether the cooperative clusters can form with time evolution, we display a series of spatial patterns under different migration costs and variance of risk preference in Fig. 7. Obviously, the cooperative clusters can be observed in all cases at a steady state. From Fig. 7, we can also know that the evolution process is as follows. The cooperation level sharply deteriorates because of strategy learning, and only a small number of cooperative clusters remain in the initial evolution stage (t=1). Then, the cooperation level rises rapidly due to the migration mechanism based on strategy, migration cost and risk preference (t=20 and t=40). Finally, the cooperators almost dominate the population at



**Fig. 4.** Probability density distribution at different variance of risk preference  $\sigma$ . Other parameters:  $\rho = 0.75$ .

the steady state (t=100). Given that  $\sigma$  = 1, a slight decrease in the number of cooperators due to the increase in migration cost (from c=1 to c=50) can be observed in the first two rows at the steady state of Fig. 7. When  $\sigma$ =50, the reduced cooperators are too small to be seen with the naked eye. These results are consistent with Fig. 5(b). Compared to the first and last row in Fig. 7, we can find that  $f_c$  is decreased slightly with the increase in variance  $\sigma$ , given the same migration cost. Observed from Fig. 7, there are three types of distribution for cooperators and defectors at t=20 and t=40, including the large cooperative clusters, the small defective clusters and a mixed strategy. In addition to the movement mechanism-based strategy, only agents around defectors can move. In summary, the migration rate of the small fraction of risk-seeking defectors is increased by the increase in variance  $\sigma$ , leading to a slight decrease in the cooperation level.

As shown in Fig. 5, we can know that the effect of the migration cost on the evolution of cooperation at b = 1.4 is different from b = 1.2. Therefore, we present the spatial pattern over time when b = 1.4 in Fig. 8. It is observed that the time required for cooperation to reach the steady state is extended when b = 1.4. Only when  $\sigma = 1$  and c = 50 is the cooperation level high. And at other values, the evolutionary behavior is unstable. We can also see that the cooperation level has increased with the increase of the migration cost compared with the first rows of Fig. 8, which is in contrast to b = 1.2. Overall, the main reason for the opposite result when b = 1.2 and b = 1.4 is the difference between the spatial pattern at the early evolution. More cooperators survive in the spatial structure of the former, while more defectors exist for the latter. In addition, for the same migration cost (c = 50), the number of cooperators is clearly reduced with the increase in variance from  $\sigma = 1$  to  $\sigma = 50$ , which can be observed by comparing the last two rows at the steady state (t = 5000) in Fig. 8. This result is consistent with Fig. 3. Overall, a series of spatial patterns is more intuitive for us to understand cooperative behavior.

We mainly give the defection parameter as b = 1.2 and b = 1.4 in the above studies. Why do we take these two values as representatives? Next, we investigate the effect of the defection parameter on the cooperation level in Fig. 9. In order to meet the conditions T > R > P > S and 2R > T + S, the defection parameter scope is 1 < b < 2. As depicted in Fig. 9,  $f_c$  as a



**Fig. 5.** The effects of migration cost on the evolution of cooperation for different mutation rates q. (a-c). q = 0.0, (d-f). q = 0.001. Other parameters: L = 40,  $\rho = 0.75$ . The data points are obtained in the same way as in Fig. 1.



**Fig. 6.** Fraction of cooperators  $f_c$  as a function of migration cost and variance of the risk preference for different defection parameter *b*. (a) b = 1.2, (b) b = 1.4. Other parameters:  $\rho = 0.625$ , q = 0.001, N = 1000. The data points are obtained in the same way as in Fig. 1.

function of *b* is a staircase function and there exists a critical value  $b^*$  making the cooperation level dramatically drop to 0 and satisfy  $b < b^*$ ,  $f_c > 0.9$  and  $b > b^*$ ,  $f_c = 0$ . As shown in Fig. 9, the critical value  $b^*$  is from about 1.4 to 1.45, and its concrete values depend on the migration cost, heterogeneity of risk preference and mutation rate. Actually, we did have a robust test on other defection parameter values in Fig. 10, and the result is similar with b = 1.2 and b = 1.4. Therefore, b = 1.2 and b = 1.4 can represent other defection parameters with which to study the evolution of cooperation.

To further verify the conclusions drawn from Fig. 9, Fig. 10(a) shows the equilibrium fraction of cooperators  $f_c$  in dependence on b and  $\sigma$ . Fig. 10(b) describes the various situation of  $f_c$  in dependence on b and c. As evidenced in Fig. 10(a) and (b), the cooperation level is relatively large at the left of the image when  $b < b^*$ , where  $b^* \in (1.4, 1.45)$ , while cooperation is difficult to achieve at the right of the image when  $b > b^*$ , irrespective of c and  $\sigma$ . Besides, a decrease in  $f_c$  with the increase of  $\sigma$  can be seen from Fig. 10(a), especially for 1.3 < b < 1.45. It can be also observed from Fig. 10(b) that  $f_c$  decreases with the increase of migration cost (c) when b < 1.35, while  $f_c$  rises with the increment of c when 1.35 < b < 1.45. The discussion about  $\sigma(c)$ -b in Fig. 10 further validates the conclusions obtained from Figs. 6 and 9.



**Fig. 7.** Some typical snapshots for different *c* and  $\sigma$ . Other parameters: q = 0.001,  $\rho = 0.75$ , b = 1.2. Risk-seeking cooperators are represented by green patches, risk aversion cooperators are represented by yellow patches, and defectors are represented by red patches. No agents are represented by white patches. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



**Fig. 8.** Some typical snapshots for different *c* and  $\sigma$ . Other parameters: q = 0.001,  $\rho = 0.75$ , b = 1.4. Risk-seeking cooperators are represented by green patches, risk aversion cooperators are represented by yellow patches, and defectors are represented by red patches. No agents are represented by white patches. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Except for the defection parameter, population density is another important factor affecting cooperation. Fig. 11 presents the effect of the population density ( $\rho$ ) on the evolution of cooperation. Two important results can be observed in Fig. 11. First, there is an optimal density value(s) at which the cooperation level is the highest, regardless of whether there is a mutation strategy. Moreover, the optimal density value is neither too large nor too small, as too large or too small density values indicate that the spatial structure is too full or too empty. Too full is not beneficial for migration of the agents, and too empty is not beneficial for the formation of cooperative clusters. Second, the region of population density for the high



**Fig. 9.** The effect of defection parameter *b* on cooperation level. (a). q = 0.0 (b). q = 0.001. Other parameters:  $\rho = 0.75$ .



**Fig. 10.** The various situation of the fraction of cooperators  $f_c$  in dependence on different parameters. (a)  $f_c$  as a function of  $(b, \sigma)$ . Other parameters:  $\rho = 0.75$ , q = 0.001, c = 50. (b)  $f_c$  as a function of (b, c). Other parameters:  $\rho = 0.75$ , q = 0.001,  $\sigma = 1$ .



**Fig. 11.** Fraction of cooperators  $f_c$  as a function of  $\rho$  for different defection parameters b. (a) q = 0.0, (b) q = 0.001.



**Fig. 12.** The various situation of the fraction of cooperators  $f_c$  in dependence on different parameters. (a)  $f_c$  as a function of  $(\rho, \sigma)$ . Other parameters: b = 1.2, q = 0.001, c = 1. (b)  $f_c$  as a function of  $(\rho, c)$ . Other parameters: b = 1.2, q = 0.001,  $\sigma = 1$ .

cooperation level is larger at b = 1.2 than at b = 1.4, regardless of the mutation rate. For q = 0, b = 1.2, we can see that the cooperation level is over 0.9 when  $\rho \in (0.5, 0.95)$ , given  $\sigma = 1$ , c = 1. A very high cooperation level is also obtained for  $\rho \in (0.5, 0.95)$  when  $\sigma = 10$ , c = 1 and  $\sigma = 10$ , c = 50. Nevertheless, for q = 0, b = 1.4, only  $\rho = 0.7$  makes the cooperators survive when  $\sigma = 1$ , c = 1. With the increase of the  $\sigma$  and c values, the region of the high cooperation level is becoming larger. Given c = 1, the increase of  $\sigma$  from 1 to 10 makes the region for high cooperation level enlarge from  $\rho = 0.7$  to  $\rho \in (0.6, 0.75)$ ; the increase in the number of risk-seeking cooperators with the increase in  $\sigma$  value improves the chances for expanding the cooperative clusters, enlarging the region of population density for the high cooperation level.

Refer to Fig. 12, the fraction of cooperation  $f_c$  as a function of  $(\rho, \sigma)$  is clearly shown in Fig. 12(a) and  $f_c$  as a function of  $(\rho, c)$  is shown in Fig. 12(b). For any  $\sigma$  and c, there is an optimal density value which is markedly more effective in raising the cooperation level. The maxima of cooperation level  $f_c$  occur at different  $\rho$  for different  $\sigma$  and c. The discussion about Fig. 12 will further prove the influence resulting from  $\rho$ . Another interesting and important finding is that an appropriate  $\sigma$  and c values will make cooperators still survive at low population density, which can be observed from Fig. 12(a) and (b). Appropriate  $\sigma$  and c values adjust the migration rate of cooperators, facilitating the formation of cooperators clusters. Besides,  $f_c$  at optimal density value is higher than at  $\rho = 1$  (without migration), indicating that the migration mechanism based on risk preference is effective for promoting cooperation.

Except for the evolution of cooperation based on the migration mechanism, how has the network structure changed from the initial moment until the steady state? Fig. 13 depicts the histogram for distribution of degree at the initial state and steady state for different population densities. Given von Neumann on the square lattice, when the population density is 1, the degree of each node is 4. However, when the population density is less than 1, the possible values of the degree for all nodes are 0 (isolated agent), 1, 2, 3 and 4. Compared with Fig. 13(a) and (b), we can see that the degree of the node gaining 4 at the steady state is more than the initial state; more cooperative clusters can be formed with individuals' migration. A similar result can be found in Fig. 13(c) and (d). Therefore, we can say that the migration mechanism based on heterogeneity of risk preference realizes the co-evolution of the network structure and cooperation strategy, which provides widely view for understanding the emergency of cooperation.

#### 4. Discussion

The effect of migration based on risk preference style on the evolution of cooperation in spatial PDG has been discussed. In previous studies, the risk preference attribute of the agent had been assumed homogeneous or even never considered. To address this, we introduce the risk appetite into the migration model and discuss the role of the homogeneous and heterogeneous risk preference on the cooperation level. Simulation results show that the migration mechanism based on risk preference provides an effective mechanism for the emergency and promotion of cooperative behavior under certain conditions. Specifically, under the homogeneous risk preference assumption and smaller defection parameter, simulation results demonstrate that the cooperation level can be raised when the whole population is risk seeking; under the larger defection parameter, the lower risk aversion values maintain the high level of cooperation. Under the heterogeneous risk preference assumption, simulation results indicate that the cooperation strategy is a winning strategy with the system approaching an evolutionary stable state. This is because the higher migration rate of risk-seeking cooperators leads to escape from invasion by defectors, which is beneficial for the expansion of cooperative clusters. This model provides realistic perspective for us to understand the evolution of cooperation.

In the analysis of the effect of migration cost and the variance of risk preference on the cooperation level, we find two important results. First,  $f_c$  increases with the increase in migration cost when b = 1.4. However, the cooperation level at b = 1.2 slightly decreases with the increase in migration cost, regardless of mutation rate q. Second,  $f_c$  shows the declining



Fig. 13. Histogram for distribution of degree at different evolution period. Other parameters:  $\sigma = 1$ , q = 0.001, b = 1.2, c = 1.

trend with the variance of the risk preference increasing. The higher migration rate for risk-seeking defectors leads to invasion by cooperators, reducing the cooperation level. Population density is another important factor affecting the evolution of cooperation. We find that there is an optimal density value(s) that makes the cooperation level reach the highest level. This result may provide a theoretical basis for policy makers. Finally, we conduct robust tests on the parameters to ensure the stability of the conclusion in our context.

Besides the evolution of cooperation strategy, the network structure based on the migration mechanism has also changed from the initial moment to the stable state. We can find that the degree of the node gaining 4 at the steady state is more than the initial state because of more cooperative clusters forming with the individuals' migration. For a more intuitive understanding, we display a series of spatial patterns under different evolution periods. The cooperative clusters and the evolution of network structure can be found in the spatial pattern at different evolution steps. We hope our current study can provide an effective way for understanding the emergence of cooperation in our society.

As long as the risk preference style is exogenously given in an initial period, the attribute of risk preference has not evolved as the strategy has evolved. Although the risk preference styles of agents are fixed at a certain period of time, it may be changed for certain reasons over time. Supposing that the risk preference style is endogenous, can the cooperation and risk preference co-evolve? Making the risk preference endogenous is an important direction of further research and is necessary to comprehensively investigate the evolution of cooperation based on migration.

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